

The University of Chicago

ASTROMETRIC AND PHOTOMETRIC STATISTICS OF
CERTAIN OF HAGEN'S FIELDS PHOTO-
GRAPHED WITH THE 24-INCH
REFLECTOR

A DISSERTATION

SUBMITTED TO THE FACULTY
OF THE OGDEN GRADUATE SCHOOL OF SCIENCE
IN CANDIDACY FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

DEPARTMENT OF ASTRONOMY AND ASTROPHYSICS

BY

HARRIET McWILLIAMS PARSONS

Reprinted from
Publications of the YERKES OBSERVATORY, Vol. IV, Part VII

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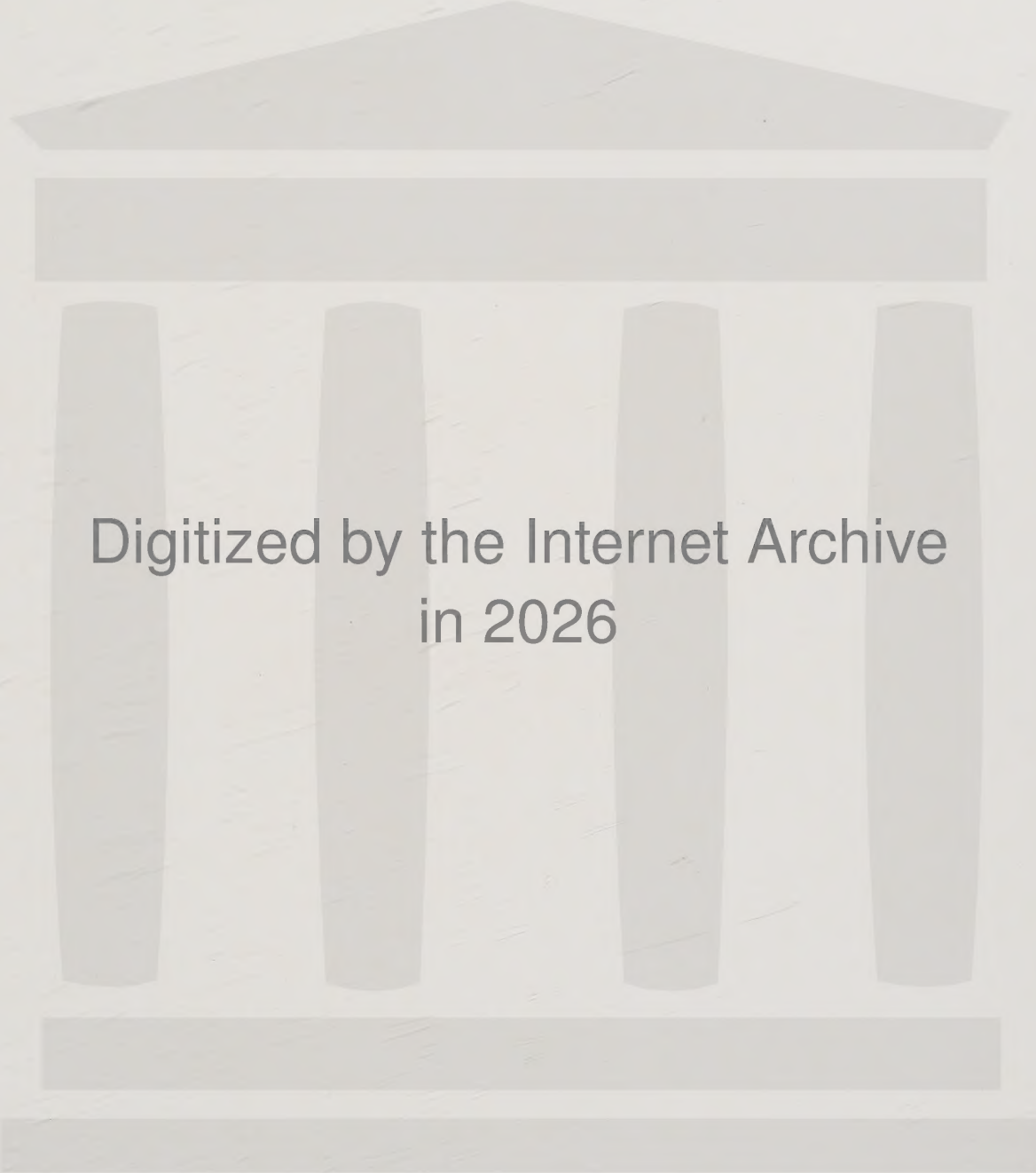
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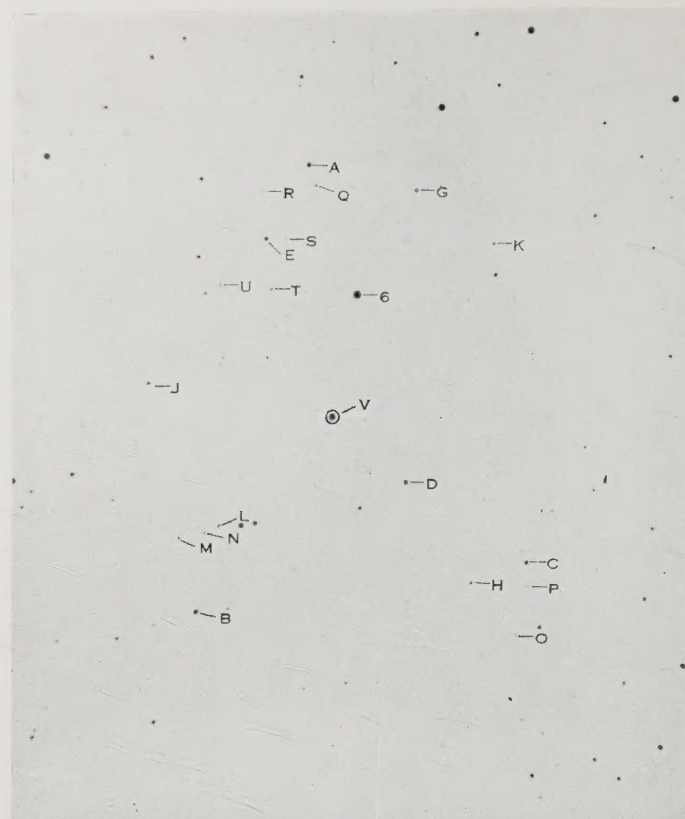
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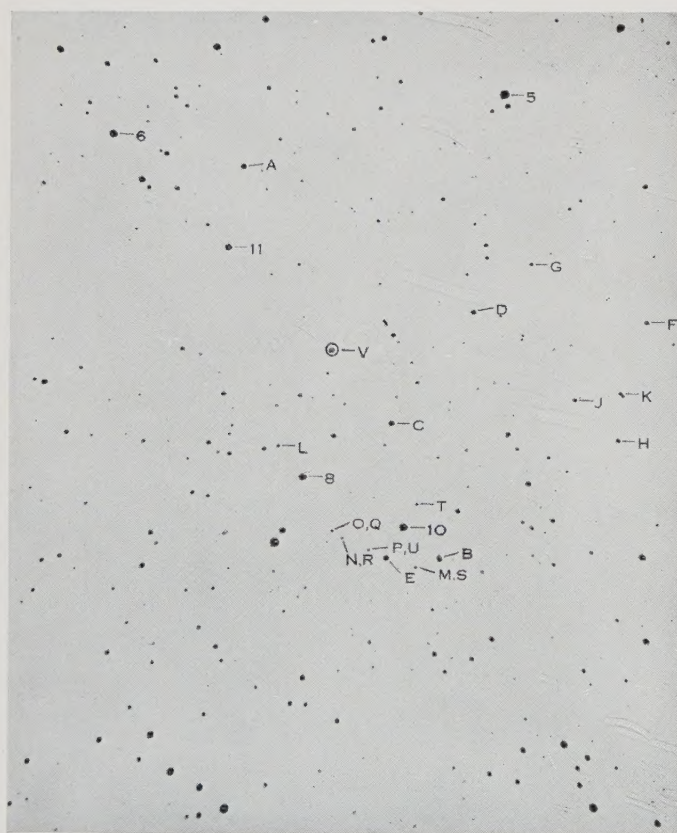
S Cassiopeiae $1^h 12^m 3, +72^\circ 5'$ (1900)
Yerkes negative, 1902, Jan. 4.

Scale, 1 mm = $19''.6$



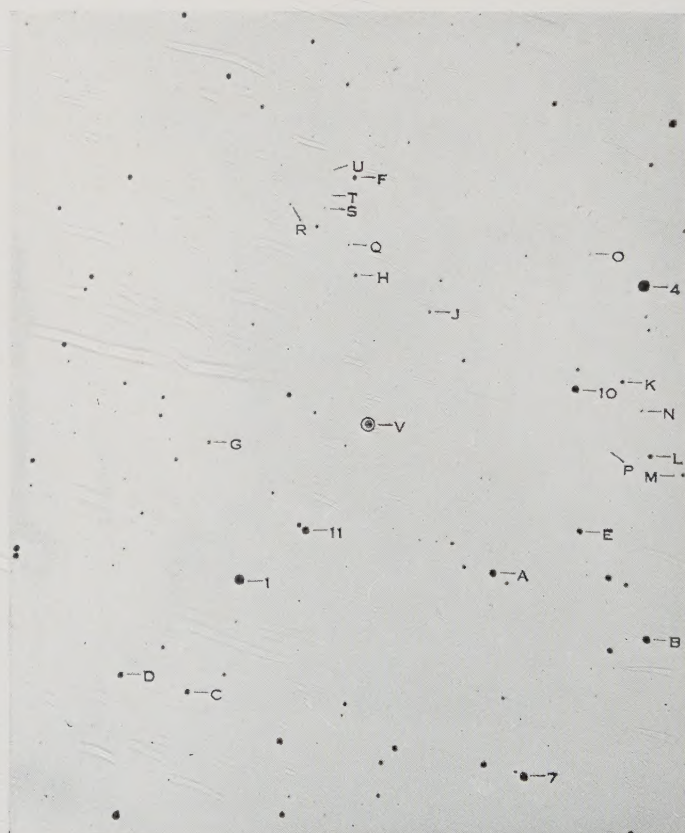
U Arietis $3^h 5^m 5, +14^\circ 25'$ (1900)
Yerkes negative, 1905, Feb. 6.

Scale, 1 mm = $20''.0$



V Tauri $4^h 46^m 2, +17^\circ 22'$ (1900)
Yerkes negative, 1902, Feb. 23.

Scale, 1 mm = $21''.7$



R Camelopardalis $14^h 25^m 1, +84^\circ 17'$ (1900)
Yerkes negative, 1902, Nov. 29.

Scale, 1 mm = $19''.9$

ASTROMETRIC AND PHOTOMETRIC STATISTICS OF CERTAIN OF HAGEN'S FIELDS PHOTOGRAPHED WITH THE 24-INCH REFLECTOR¹

BY HARRIET MCWILLIAMS PARSONS

INTRODUCTION AND GENERAL DESCRIPTION

The object of this investigation is the determination of positions, proper motions, and magnitudes, both photographic and photovisual, of comparison stars in eight of the fields in Hagen's *Atlas Stellarum Variabilium*, from photographs made with the 24-inch reflector of the Yerkes Observatory. A study of the interrelations of the various results is also included.

The choice of these eight fields is a result of the following circumstances. In 1900, Professor E. C. Pickering, of the Harvard College Observatory, selected thirty-six of Hagen's variable-star fields for special research.² In each of these regions the variable is of long period and is very faint at minimum. The following set of comparison stars was chosen in each field, the visual magnitudes of which were to be determined by observers at the Harvard College Observatory, the Halsted, the McCormick, the Lick, and the Yerkes observatories.

Five twelfth-magnitude stars, *a, b, c, d, e*

Five fifteenth-magnitude stars, *l, m, n, o, p*

Five fourteenth-magnitude stars, *f, g, h, i, k*

Five sixteenth-magnitude stars, *q, r, s, t, u*

Grants from the Rumford fund were made to the observers undertaking the determination of the magnitudes of these stars, and the fields were therefore called "Rumford regions."

During the summers of 1918 and 1919, I measured positions of the twelfth-, fifteenth-, and sixteenth-magnitude selected comparison stars in the thirty-six Rumford regions from plates taken with the 24-inch reflector, at the suggestion of Professor Parkhurst. The same instrument which was used in the subsequent work described here was employed. In most of these regions plates had been taken with this telescope soon after the completion of the instrument, about 1900, and these proved to be satisfactory for astrometric purposes. Two plates were retaken because the earlier plates did not show all of the sixteenth-magnitude stars. For determining the constants of reduction for each plate, stars whose positions are given in the *Catalog der Astronomischen Gesellschaft* were used as standards, five stars being chosen for each field. Since only the brighter stars are included in this *Catalog*, and these stars in most cases are too far from the optical axis to give good images with the reflector, the constants as determined from these stars were not sufficiently accurate to give satisfactory positions. Ten of the northern regions are included in the *Astrographic Catalogue*, with stars near the center of the plate in sufficient number for the determination of more accurate constants. These ten regions were chosen for special study, both astrometric and photometric. One field, R Aquilae, was later omitted, because the faintest selected stars required at least two and one-half hours exposure time, and W Leonis was also omitted.

All of the plates used in this work were taken with the Yerkes 24-inch reflector. This telescope³ has a rigid and very heavy mounting especially designed for photographic work by Wadsworth with modifications by Ritchey, and constructed in the instrument shop of the Observatory. The concave silvered-glass mirror is 59.7 cm ($23\frac{1}{2}$ in.) in diameter and has a focal length of 236.5 cm (93 in.). The mounting is of the Newtonian type. The image is formed on a plate near the upper end of the

¹ A dissertation submitted to the Faculty of the Ogden Graduate School of Science of the University of Chicago in candidacy for the degree of Doctor of Philosophy, 1921.

² "Photometric Magnitudes of Faint Standard Stars Measured Visually at Harvard, Yerkes, Lick and McCormick Observatories," prepared for publication by S. A. Mitchell, *Memoirs of the American Academy of Arts and Sciences*, **14**, 209-307 (Part IV, 1923). The half-tone blocks used here, by permission, were prepared for and printed in that Memoir.

³ *Astrophysical Journal*, **14**, 217, 1901; Hale, *Study of Stellar Evolution*, p. 43.

tube after reflection of the cone of light from a small plane mirror placed at an angle of 45° to the optical axis. The double-slide plate carrier, which holds $3\frac{1}{4} \times 4\frac{1}{4}$ plates, is similar to that used with the 40-inch refractor. The field photographed is 7.5 cm (3 in.) square. The plate holder is carried on two slides at right angles to each other, and is moved by means of two screws operated by the observer. An eyepiece mounted on the carrier just outside of the field to be photographed can be moved in one co-ordinate, and the whole carrier can be rotated in such a way that a suitable guiding star can be found. Cross-wires are illuminated by light from a small incandescent lamp, passing through a red filter. Most of the corrections needed in guiding are due to slight errors in the rate of the clock. With the carrier in a normal position, i.e., with the motions of the screws in right ascension and declination, respectively, the main part of the guiding can be done by means of the screw moving in right ascension. However, errors and changes in declination must be watched for carefully and corrected by means of the other screw.

Before taking a plate, the proper focus is found by a Foucault knife-edge test, using the image of a bright star for this purpose. This focus, as determined, must be increased by twenty divisions (0.88 mm or 0.0347 in.), for isochromatic plates employed with the yellow color filter J1, now in use, because of the refraction of the light passing through the glass filter.

When the plate holder is removed an eyepiece is used in finding the field. The variable star at the center of the region to be photographed is brought to the intersection of the cross-wires in this eyepiece, when the readings of the scales indicate the proper collimation of the instrument. The proper collimation of the 24-inch reflector has been discussed recently by Miss Farnsworth.¹

In most of this work Seed 30 plates were used. Some of the earlier plates were Cramer Crown, while for the determination of photovisual magnitudes, Cramer Isochromatic plates and the yellow filter were used. The exposures range from ten to one hundred minutes, the majority being of about sixty-minute duration.

PART I. ASTROMETRIC INVESTIGATION

Early plates of the eight Rumford fields investigated were taken between the years 1901 and 1908. For the determination of proper motions, later plates were made of these fields between September, 1919, and June, 1921. These later exposures were made at practically the same hour angle as the early plate, with a similar aperture and exposure time, and, wherever possible, the same brightness of the variable. Table I gives a summary of these plates.

The plates were measured on the new screw measuring machine of the Yerkes Observatory.² This instrument is built for 8×10 plates, so a special arrangement for $3\frac{1}{4} \times 4\frac{1}{4}$ reflector negatives was necessary. The most satisfactory arrangement and the one used in most of this work is a plate-carrying kit, made in the instrument shop of the Observatory by Mr. McElwain. The kit was clamped to the measuring machine, and the negative was fastened to the kit with clips.

The plate was oriented by means of two stars whose positions are given in the *Astrographic Catalogue*, differing as much as possible in one co-ordinate and as little as possible in the other. If A and B represent the positions of the two stars, the line AC represents the ideal north and south line on the plate or the difference in the astrographic y co-ordinates of the stars, and BC represents the difference in the astrographic x co-ordinates, then the tangent of angle $CAB = \frac{\Delta x}{\Delta y}$. By means of the orientation screw of the measuring machine, the kit containing the plate was turned until the stars A and B were bisected by the vertical spider line in the eyepiece as each image was brought in turn to the center of the field of view. Angle CAB was computed from the astrographic positions, and the kit was turned through the required angle by means of the orientation screw until all points

¹ *Popular Astronomy*, 29, 207, 1921.

² Lee and Steele, "An Investigation of a New Screw Measuring Machine at the Yerkes Observatory," *Publications of the Yerkes Observatory*, 4, 65, 1917.



R Serpentis $15^{\text{h}} 46^{\text{m}} 1, +15^{\circ} 26'$ (1900)
Yerkes negative, 1904, June 18.

Scale, 1 mm = 19"7



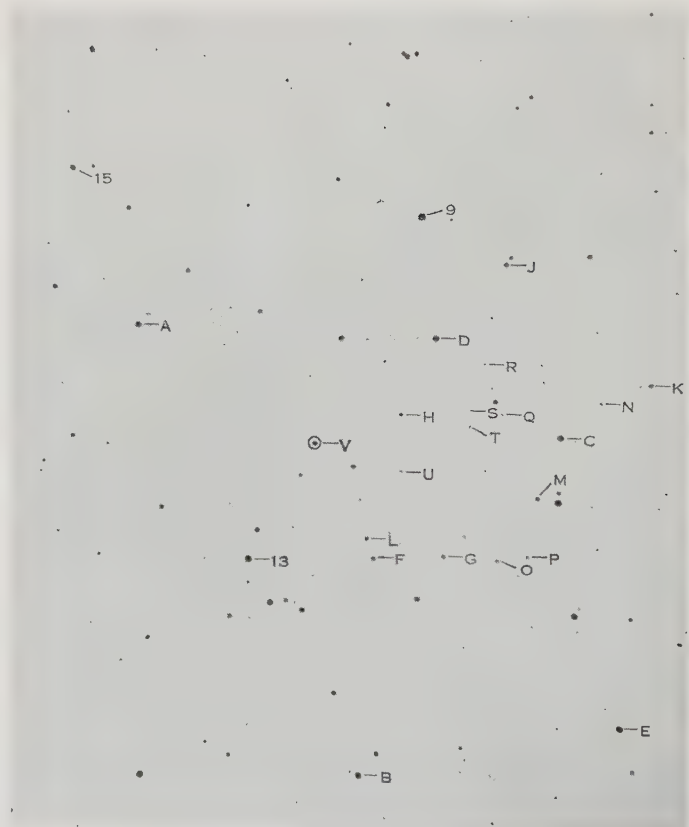
R Draconis $16^{\text{h}} 32^{\text{m}} 4, +66^{\circ} 58'$ (1900)
Yerkes negative, 1915, Oct. 1.

Scale, 1 mm = 19"6



R Vulpeculae $20^{\text{h}} 59^{\text{m}} 9, +23^{\circ} 26'$ (1900)
Yerkes negative, 1915, Nov. 2.

Scale, 1 mm = 23"0



R Pegasi $23^{\text{h}} 1^{\text{m}} 6, +10^{\circ} 0'$ (1900)
Yerkes negative, 1901, Dec. 29.

Scale, 1 mm = 17"7

TABLE I
ASTROMETRIC PLATES

Field	Plate Numbers Series R	Date	Kind of Plate	Aper. in Inches	Hour Angle Begin	Exp. Time	Mag. of V	Quality
		yr.						
S Cassiopeiae	229	1905.729	Seed 27	18	$-3^h 53^m$	60 ^m	13.8	good
$\alpha = 1^h 12^m 3$	4011	1919.717	Seed 30	17	$-3 \ 51$	60	<14.5	good
$\delta = +72^\circ 5'$	4018	1919.728	C.I.M.	17	$-3 \ 47$	90	<14.5	fair
U Arietis.	193	1905.102	Seed 27	19	$+1 \ 3$	60	9.8	good
$\alpha = 3^h 5^m 5$	4081	1920.111	Seed 30	18	$+1 \ 7$	60	14.0	good
$\delta = +14^\circ 25'$								
V Tauri.	19	1902.148	C. Iso.	24	$+3 \ 27$	65	10.2	moderately good
$\alpha = 4^h 46^m 2$	4242	1921.168	Seed 30	24	$+3 \ 13$	65	10.2	good
$\delta = +17^\circ 22'$								
R Camelopardalis	2070	1908.226	Seed 27	18	$-2 \ 33$	60	8.8	good
$\alpha = 14^h 25^m 1$	4274	1921.275	Seed 30	17	$-2 \ 47$	64	<14.8	fair
$\delta = +84^\circ 17'$								
R Serpentis.	63	1904.464	C. Crown	19	$-0 \ 06$	90	> 8.5	good
$\alpha = 15^h 46^m 1$	4264	1921.258	Seed 30	17	$-0 \ 06$	90	8.6	moderately good
$\delta = +15^\circ 26'$								
R Draconis.	59	1904.445	C. Crown	19	$-1 \ 37$	100	13. —	fair
$\alpha = 16^h 32^m 4$	4291	1921.349	Seed 30	17	$-1 \ 37$	100	13. —	p-f
$\delta = +66^\circ 58'$								
R Vulpeculae	130	1904.831	Seed 27	22	$+1 \ 4$	60	11.7	{good
$\alpha = 20^h 59^m 9$	3283	1915.837	Seed 30	24	$+0 \ 57$	90	11.7	{very strong
$\delta = +23^\circ 26'$	4021	1919.733	Seed 30	17	$+1 \ 48$	60	9.5	{good
	4034	1919.862	Seed 30	22	$+2 \ 20$	50	11.5	{moderately good
	4204	1920.930	Seed 30	22	$+1 \ 45$	60	11.7	{fair
								{fair
R Pegasi	10	1901.994	C. Crown	24	$+2 \ 28$	90	12.5	good
$\alpha = 23^h 1^m 6$	4208	1921.015	Seed 30	24	$+2 \ 1$	55	11.0	fair
$\delta = +10^\circ 0'$								

on the line AC fell on the vertical wire as they were moved into the center of the field. The amount of rotation can be read to 0.1 with the vernier.

The X co-ordinate for each star was read by moving the eyepiece carrying the spider lines, until the vertical wire and the center of the star image coincided, always moving the screw in the same direction. The screw head may be read directly to 0.001 mm, and estimated to the next decimal place. The stars measured in each field were the variable, the stars suitable for standards in determining the plate constants, and the twelfth-, fifteenth-, and sixteenth-magnitude selected stars. Three settings were made on each image. By means of the kit the plate could be turned through 90° , 180° , 270° , with a probable error of $10''$ in angle. The plates were measured in four positions, direct and reversed in both X and Y .

The mean distance of each star from the variable was found in X and Y , expressed in millimeters. Multiplying these co-ordinates by 87.38 reduced the millimeters approximately to seconds of arc.

Using as standard stars those, not more than $15'$ from the variable, whose positions are given in the *Astrographic Catalogue*, the six plate constants were determined: p and p' , the corrections to the scale value in X and Y ; r and r' , orientation corrections; and k and c , corrections to the position of the variable in the center of the plate. Table II gives the following astrographic data: the Rumford field contained on each plate,

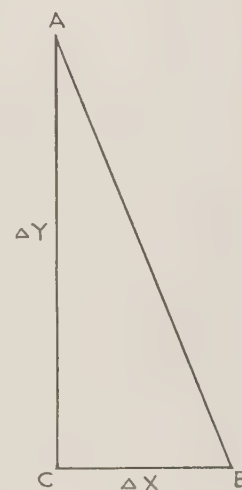


FIG. 1

the observatory publishing the work, the plate number, and date. By Turner's method of reduction,¹ if x and y represent the ideal co-ordinates of a star from the center of the plate, and X and Y represent the measured co-ordinates, we have

$$\begin{aligned}x &= X + pX + rY + k, \\y &= Y + p'Y - r'X + c.\end{aligned}$$

The observation equations are

$$\begin{array}{ll}pX_1 + rY_1 + k - X_1 = 0 & p'Y_1 - r'X_1 + c - Y_1 = 0 \\ \vdots & \vdots \\pX_n + rY_n + k - X_n = 0 & p'Y_n - r'X_n + c - Y_n = 0\end{array}$$

The normal equations are then²

$$\begin{array}{ll}[XX]p + [XY]r + [X]k = [X\Delta X] & [YY]p' - [XY]r' + [Y]c = [Y\Delta Y] \\[XY]p + [YY]r + [Y]k = [Y\Delta X] & [XY]p' - [XX]r' + [X]c = [X\Delta Y] \\[X]p + [Y]r + nk = [\Delta X] & [Y]p - [X]r + nc = [\Delta Y]\end{array}$$

TABLE II
ASTROGRAPHIC PLATES

Field	Observatory	Plate No.	Date			Date
			yr.	mo.	da.	yr.
S Cassiopeiae	Greenwich	4601	1899	8	9	1899.607
U Arietis	Bordeaux	993	1906	1	23	1906.063
V Tauri	Bordeaux	552	1901	2	11	1901.115
R Camelopardalis	Greenwich	4541	1899	6	16	1899.459
R Serpentis	Bordeaux	573	1901	6	20	1901.469
R Draconis	Greenwich	2652	1895	6	2	1895.421
R Vulpeculae	Paris	737	1894	9	12	1894.700
R Pegasi	Toulouse	173	1897	10	28	1897.827

The effect of the plate constants may be explained as follows: p and p' represent corrections to the scale value, showing that the quantity 87".38 per millimeter is not exactly correct for every plate, according to the astrographic system. The effect of refraction enters into these values, and temperature changes in the scale of the plate, if they exist. If these quantities were applied alone, the equation would be

$$\begin{aligned}x &= X + pX, \\y &= Y + p'Y,\end{aligned}$$

p and p' should not differ greatly on any one plate, and when the means are taken for several plates they should be practically identical.

The orientation corrections r and r' are due to the fact that it is impossible to turn through an angle with sufficient accuracy. When these constants are applied to a plate its orientation is changed to that of the astrographic plate from which the standard positions were obtained. If θ represents the orientation error, taken in direction of decreasing position angle,

$$\begin{aligned}x &= AC & y &= SC \\X &= AD & Y &= SD \\x &= AC = AE \cos \theta \\AE &= AD + DE = X + Y \tan \theta \\x &= X \cos \theta + Y \sin \theta.\end{aligned}$$

¹ *Monthly Notices*, 54, 11, 1893.

² Campbell, *Elements of Practical Astronomy*, 1906.

If θ is small,

$$\cos \theta = 1$$

$$\sin \theta = \theta = r$$

$$x = X + Yr$$

$$y = SC = SB \cos \theta$$

$$SB = SD - DB = Y - AD \tan \theta = Y - X \tan \theta$$

$$y = Y \cos \theta - X \sin \theta$$

$$y = Y - Xr'.$$

The corrections to the zero point, k and c , are due to the fact that the variable may not be at the position given in the *Catalogue* on the plate measured. Included in these values are errors which arise in the measurement of the variable, especially if it is very faint or very bright, and errors in the *Catalogue* positions. When these values are applied to early and late plates, the results will give the distances of the comparison stars from an ideal position—that of the variable in the *Astrographic Catalogue*. Any proper motion of the variable will not, then, affect the final results for the other stars.

Comparing the measured positions of the stars and the positions as given in the *Astrographic Catalogue*, the values for p , p' , r , r' , k , and c were determined by least-square solutions, and applied to the measured positions. Each plate was measured twice at an interval of several months. Table III tabulates these results. The factors p and p' are of the same order in each case (see Table IV), averaging -0.002 . The value $87''.38$ per millimeter might be changed by $87''.38 \times (-0.002)$ or $(-0''.17)$ leaving $87''.21$ per millimeter for the scale value of reflector plates derived from astrographic positions. The corrections show an increase arithmetically as the aperture of the telescope is increased (Table V). Since p and p' are negative, showing that $87''.38$ per millimeter is too large on the astrographic basis, the arithmetic increase in p with the aperture would indicate that my tendency in measuring is to bisect the images on 24-inch plates too far from the center of the field. The researches of Miss Farnsworth and Mr. Hubble on the field of the 24-inch reflecting telescope are of interest in this connection. Miss Farnsworth's work on 18-inch plates, (1) from comparison of scale values obtained from stars at different distances from the axis, (2) from changes in length of a measured interval as it took up positions at different distances from the axis, indicated an increasing scale value with increasing ρ when the settings are on the nuclei, as if the settings were too near the axis. She states that no correction seems necessary within $20'$ – $25'$ in the 18-inch field where the images are bisected.¹ Her work is on a slightly different basis from that of this paper where a comparison of different apertures is being made. Mr. Hubble's work indicates the difficulty of measuring images not directly at the center of the plate. "With the full aperture, the stellar images are sensibly round only within $5'$ of the optical axis."² Since this distortion exists for 24-inch plates within the region in which the stars chosen for standards exist, the increase in p and p' with increasing aperture is explained. It is important, therefore, to compare plates taken with the same aperture if any determination of proper motion is desired.

Miss Young, of Mount Holyoke College Observatory, has determined the scale of the reflector

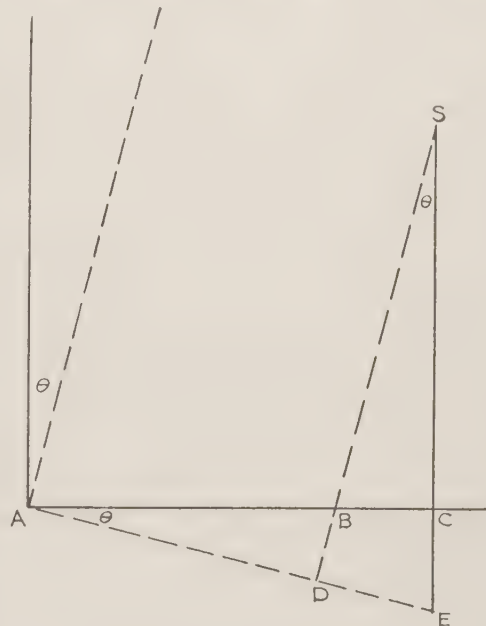


FIG. 2

¹ *Publications of the Yerkes Observatory*, 4, 191, 1926.

² *Ibid.*, p. 69, 1920.

plates to be 87".38 per millimeter from measures of stars from the *Astronomische Gesellschaft Catalog*. This value is the one used as a provisional scale value in my reductions. My mean value from astrographic standards is 87".21, as stated before. This value is greater for plates taken with 18-inch aperture, and less for plates with 24-inch aperture. The stars used by Miss Young are mainly

TABLE III
PLATE CONSTANTS

Field	Plate	p	p'	r	r'	k	c	Aper. in Inches
S Cassiopeiae.....	229	-0.0013	-0.0013	+0.0005	+0.0008	-0".674	+0".278	18
	229	.0016	.0016	- .0004	- .0011	- .427	+ .161	
	4011	.0014	.0020	- .0000	- .0005	- .832	+ .789	17½
	4018	.0017	.0018	+ .0005	+ .0007	- .742	+ .356	
U Arietis.....	193	.0019	.0021	- .0003	- .0003	- .141	- .094	19
	193	.0021	.0020	- .0006	- .0009	+ .003	- .066	
	4081	.0021	.0020	+ .0004	- .0001	+ .605	- .047	18
	4081	.0022	.0024	+ .0002	- .0001	+ .353	- .364	
V Tauri.....	19	.0030	.0031	+ .0007	+ .0010	+ .087	- .580	24
	19	.0020	.0026	+ .0003	+ .0016	+ .353	- .449	
	4242	.0031	.0028	+ .0012	+ .0004	- .345	- .385	24
	4242	.0032	.0025	+ .0007	+ .0001	- .061	- .473	
R Camelopardalis....	2070	.0013	.0010	- .0008	- .0024	- .174	- .424	18
	2070	.0012	.0017	- .0022	- .0027	+ .175	- .398	
	4274	.0018	.0019	- .0004	- .0004	+ .503	- .579	17½
	4274	.0014	.0018	+ .0003	+ .0001	+ .195	+0.179	
R Serpentis.....	63	.0019	.0023	+ .0003	+ .0005	+ .234	-1.206	19
	63	.0017	.0022	+ .0010	+ .0016	+ .223	-0.628	
	4264	.0018	.0016	- .0007	- .0001	+ .293	-1.075	17½
	4264	.0018	.0014	+ .0008	+ .0013	+ .212	-0.958	
R Draconis.....	59	.0021	.0027	+ .0001	+ .0015	- .427	- .360	19
	59	.0016	.0022	+ .0005	+ .0011	- .585	- .318	
	4291	.0020	.0024	- .0001	+ .0003	+ .492	+ .057	17½
	4291	.0020	.0013	+ .0001	+ .0010	+ .620	+ .573	
R Vulpeculae.....	130	.0021	.0029	- .0003	- .0010	- .140	- .141	22
	130	.0020	.0031	- .0003	- .0001	- .487	- .068	
	3283	.0031	.0034	- .0005	- .0000	- .335	- .032	24
	3283	.0038	.0037	+ .0005	+ .0004	+ .029	- .083	
	4021	.0016	.0019	+ .0008	+ .0013	- .342	- .204	17½
	4021	.0015	.0020	+ .0006	+ .0012	- .194	+ .006	
	4034	.0028	.0031	- .0004	+ .0000	- .658	- .031	22
	4034	.0027	.0032	+ .0003	+ .0011	- .492	+ .173	
	4204	.0036	.0041	+ .0009	+ .0017	-0.533	+ .140	22
	4204	.0029	.0039	- .0047	+ .0016	-1.854	+ .218	
R Pegasi.	10	.0030	.0031	- .0007	- .0011	+0.038	- .300	24
	10	.0027	.0029	- .0005	- .0012	+ .080	- .323	
	4208	.0037	.0020	+ .0009	- .0014	- .633	- .427	24
	4208	-0.0033	-0.0022	+0.0016	-0.0003	+0.782	-0.418	

TABLE IV
SUMMARY OF CORRECTIONS TO SCALE

Field	p	p'
S Cassiopeiae.....	-0.001500	-0.001675
U Arietis.....	.002075	.002125
V Tauri.....	.002825	.002750
R Camelopardalis.....	.001425	.001600
R Serpentis.....	.001800	.001875
R Draconis.....	.001925	.002150
R Vulpeculae.....	.002600	.003130
R Pegasi.....	-0.003175	-0.002550
Mean.....	-0.002166	-0.002232

* Mean correction to scale for all R plates = -0.002199.

far from the axis and therefore distorted. It would seem from this comparison that Miss Young measures systematically closer to the axis than I do, or that she bisects the nucleus, while my tendency is to strike a mean between the center of gravity of the image and the center of the nucleus. These tendencies in measuring would have no effect on the proper motions obtained since early and late plates are measured by the same observer.

TABLE V
RELATION BETWEEN SCALE CORRECTION AND APERTURE

Field	Aper. in Inches	p	p'
V Tauri	24	-0.002825	-0.002750
R Vulpec. (without R 4021)	22	.002600	.003130
R Pegasi	24	-0.003175	-0.002550
Mean		-0.002867	-0.002810
S Cassiopeiae	18	-0.001500	-0.001675
U Arietis	18	.002075	.002125
R Camelopardalis	18	.001425	.001600
R Serpentis	18	.001800	.001875
R Draconis	18	-0.001925	-0.002150
Mean		-0.001745	-0.001885

* Mean correction to scale for 18-inch plates = -0.001815; mean correction to scale for 24-inch plates = -0.002838; excess correction for 24-inch plates over 18-inch plates = -0.001023.

The values for r and r' differ in sign and are largely accidental, due to the fact that the stars used in orientation of the plate were often near the edge of the field, and were consequently difficult to bisect. The values for k and c show little preference for sign (Table VI). Table VII gives the astrometric results from the measurements just described. The first column gives the Hagen designation for the stars, and the l to u Rumford designations (see p. 1). Column 2 gives the astrographic x distance in seconds. Column 3 gives the x distance of each star from the variable on the early plate. Column 4 gives the average deviation from the mean of these x distances from two or more sets of measures. Columns 5 and 6 are similar to 3 and 4 for the later plates. Column 7 gives the difference between positions on the early plate and the astrographic positions, and column 8 the difference between positions on the late and early plates, thus giving two sets of quantities from which proper motions can be inferred. The next seven columns give the same data for the y distances while column 16 gives proper motions for all the stars. Stars too far from the center to give measurable images are marked questionable, using a colon. Stars used as standards are marked with an asterisk.

The majority of the proper motions from these measurements are small, many of them being less than the average deviations between separate measures of the same images on the same plates. In a few cases differences of one second or over appear, which may be considered as real. The interval between the early and late plates ranges from thirteen to nineteen years (see Table I).

TABLE VI
MEAN PLATE CONSTANTS FOR SEPARATE FIELDS

FIELD	r	r	r'	r'	k	k	c	c
	Algebraic	Arithmetic	Algebraic	Arithmetic	Algebraic	Arithmetic	Algebraic	Arithmetic
S Cassiopeiae	+0.000200	0.000350	-0.000025	0.000775	-0.6688	0.6688	+0.3960	0.3960
U Arietis	- .000075	.000375	- .000350	.000350	+ .2050	.2755	- .1428	.1428
V Tauri	+ .000725	.000725	+ .000775	.000775	+ .0085	.2115	+ .4718	.4718
R Camelopardalis	- .000775	.000925	- .001350	.000140	+ .1748	.2618	- .3055	.3950
R Serpentis	+ .000350	.000700	+ .000825	.000875	+ .2405	.2405	- .9668	.9668
R Draconis	+ .000150	.000175	+ .000975	.000975	+ .0250	.5310	- .0120	.3270
R Vulpeculae	- .000310	.000930	+ .000620	.000840	- .5006	.5064	- .0022	.1096
R Pegasi	+0.000325	0.000925	-0.001000	0.001000	+0.3832	0.3832	-0.3670	0.3670
Mean	+0.000074	0.000638	+0.000059	0.000716	-0.0165	0.3848	-0.2340	0.3970

TABLE VIIa
ASTROMETRIC RESULTS—S CASSIOPEIAE

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Des.	$\overset{x}{\text{Astr.}}$ 1899.1607	$\overset{x}{\text{R229}}$ 1905.729	Aver. Dev. from Mean	$\overset{x}{\text{R4011, 4018}}$ 1919.722	Aver. Dev. from Mean	$\overset{x}{\text{R229}}$ — $\overset{x}{\text{Astr.}}$	$\overset{x}{\text{R4011, 4018}}$ R229	$\overset{y}{\text{Astr.}}$	$\overset{y}{\text{R229}}$	Aver. Dev. from Mean	$\overset{y}{\text{R4011, 4018}}$	Aver. Dev. from Mean	$\overset{y}{\text{R229}}$ Astr.	$\overset{y}{\text{R4011, 4018}}$ R229	P.M. for 13,993 Yr.
1*	— 278.59	— 278.06	0.12	— 278.12	0.19	+0.53	—0.06	+958.18	+958.69	0.22	+958.66	0.06	+0.51	—0.03	0.07
2*	+ 1166.90	+ 1167.12	.14	+ 1167.26	.42	+ .22	+ .14	+872.65	+873.00	.23	+872.92	.23	+ .35	— .08	.16
3*	— 249.61	— 249.04	.00	— 249.37	.22	+ .57	— .33	—757.43	—757.62	.17	—757.41	.04	— .19	+ .21	.39
4*	+ 20.87	+ 20.85	.18	+ 20.63	.12	— .02	— .22	+504.67	+504.69	.03	+504.49	.10	+ .02	— .20	.30
5*	— 1471.06	— 1471.87	.29	— 1471.72	.08	— .81	+ .15	—418.64	—418.78	.14	—418.30	.12	— .14	+ .48	.50
9*	— 747.58	— 747.92	.18	— 747.66	.14	— .34	+ .26	—631.52	—631.92	.04	—631.88	.22	— .40	+ .04	.26
10*	+ 100.34	+ 100.66	.06	+ 100.02	.26	+ .32	— .64	—801.32	—801.44	.09	—801.86	.10	— .12	— .42	.44
11*	— 199.99	— 199.80	.08	— 200.06	.02	— .19	— .26	—166.10	—165.92	.07	—166.28	.07	+ .18	— .36	.73
12*	+ 299.42	+ 299.16	.18	+ 299.27	.05	— .26	+ .11	+440.41	+440.02	.06	+440.20	.06	+ .39	+ .18	.32
14*	— 867.83	— 867.78	.06	— 868.00	.06	— .05	+ .22	—50.42	—49.86	.02	—50.22	.03	+ .56	— .36	.38
20*	+ 301.55	+ 301.68	.06	+ 301.58	.08	+ .13	+ .10	+458.95	+459.11	.03	+459.25	.00	+ .16	+ .14	.26
22*	+ 557.00	+ 556.75	.04	+ 556.82	.03	— .25	+ .07	+835.63	+834.86	.35	+835.16	.06	— .77	+ .30	.32
25*	— 19.57	— 19.84	.37	— 198.16	.12	— .31	— .23	+694.45	+694.04	.06	+694.26	.06	— .41	+ .22	.23
27*	— 677.76	— 677.76	.19	— 677.94	.16	— .07	— .23	—453.13	—453.13	.00	—452.62	.18	— .51	+ .60	.60
35*	— 759.16	— 759.16	.14	— 759.32	.06	— .27	— .23	—273.44	—273.20	.04	—272.95	.27	+0.24	+ .25	.34
<i>l</i>	— 294.62	— 294.62	.07	— 294.84	.02	— .02	— .23	—97.17	—97.17	.06	—96.68	.27	— .49	+ .49	.52
<i>m</i>	+ 603.98	+ 603.98	.01	+ 603.92	.02	— .02	— .23	+379.33	+379.33	.01	+379.38	.20	— .05	+ .05	0.17
<i>n</i>	+ 638.95	+ 638.95	.03	+ 639.12	.04	— .02	— .23	+983.18	+983.18	.06	+981.96	.12	— .41	+ .22	1.24
<i>o</i>	+ 449.28	+ 449.28	.01	+ 449.54	.12	— .02	— .23	+618.26	+618.26	.00	+618.22	.04	— .04	— .04	0.08
<i>p</i>	— 375.56	— 375.56	.00	— 375.22	.07	— .02	— .23	+751.56	+751.56	.01	+751.16	.14	— .49	+ .49	0.43
<i>q</i>	+ 355.80	+ 355.80	.08	+ 355.74	.12	— .02	— .23	+831.40	+831.40	.21	+830.30	.30	— .10	+ .10	1.13
<i>r</i>	+ 453.92	+ 453.92	.10	+ 454.41	.20	— .02	— .23	+795.86	+795.86	.24	+795.30	.24	— .56	+ .56	0.65
<i>s</i>	+ 406.26	+ 406.26	0.13	+ 406.22	0.20	— .02	— .23	+808.24	+808.24	.14	+807.68	.25	— .62	+ .62	.56
<i>t</i>	— 406.26	— 406.26	0.13	— 406.22	0.20	— .02	— .23	+709.62	+709.62	.14	+709.00	.25	— .62	+ .62	.79
<i>u</i>	— 406.26	— 406.26	0.13	— 406.22	0.20	— .02	— .23	+873.22	+873.22	0.04	+872.56	0.30	— .66	+ .66	0.66

TABLE VIIb
ASTROMETRIC RESULTS—U ARIETIS

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Des.	α Astrog. 1906.083	α R193 1905.102	Aver. Dev. from Mean	α R4081 1920.111	Aver. Dev. from Mean	α R193 Astrog.	α R4081 R193	α Astrog.	α R193	Aver. Dev. from Mean	α R4081	Aver. Dev. from Mean	α R193 Astrog.	α R4081 R193	P.M. for 15,009 Yr.
2:	231.95	232.84	0.23	—	0.28	—0.89	+1.48	+2063.41	+2065.69	0.08	+2062.84	0.62	+2.28	—2.85	3.21
3:	—	—	.04	—	.02	—	—0.24	—1644.55	—1644.77	.15	—1644.44	.10	—0.22	+0.33	0.41
4:	—	—	.24	—	.12	—	—	+2080.52	+2083.16	.30	+2076.30	.54	+2.54	—6.86	6.89
5:	—	—	.04	—	.02	—	—	+1201.52	+1202.20	.10	+1201.80	.32	+0.68	—0.40	0.75
6:	—	—	.14	—	.10	—	—	—	—	.00	—	.03	—	—	.50
9*	—	—	.07	—	.04	—	—	—	—	.00	—	.01	—	—	.25
10*	—	—	.10	—	.00	—	—	—	—	.17	—	.02	—	—	.50
12*	—	—	.01	—	.10	—	—	—	—	.18	—	.04	—	—	.28
13:	—	—	.02	—	.14	—	—	—	—	.04	—	.30	—	—	1.13
14*	—	—	.10	—	.03	—	—	—	—	.06	—	.02	—	—	0.47
16*	—	—	.02	—	.23	—	—	—	—	.09	—	.00	—	—	.16
17:	—	—	.19	—	.04	—	—	—	—	.04	—	.30	—	—	.32
19:	—	—	.16	—	.21	—	—	—	—	.03	—	.19	—	—	.77
20*	—	—	.08	—	.01	—	—	—	—	.09	—	.14	—	—	.25
21:	—	—	.22	—	.01	—	—	—	—	.14	—	.14	—	—	.49
22:	—	—	.08	—	.14	—	—	—	—	.12	—	.09	—	—	.51
23:	—	—	.10	—	.02	—	—	—	—	.02	—	.16	—	—	.21
<i>l</i>	—	—	.10	—	.04	—	—	—	—	.12	—	.11	—	—	.21
<i>m</i>	—	—	.04	—	.16	—	—	—	—	.02	—	.04	—	—	.77
<i>n</i>	—	—	.07	—	.19	—	—	—	—	.16	—	.09	—	—	.36
<i>o</i>	—	—	.04	—	.06	—	—	—	—	.08	—	.06	—	—	.63
<i>p</i>	—	—	.12	—	.16	—	—	—	—	.08	—	.06	—	—	.24
<i>q</i>	—	—	.08	—	.07	—	—	—	—	.12	—	.02	—	—	.23
<i>r</i>	—	—	.09	—	.14	—	—	—	—	.12	—	.08	—	—	.49
<i>s</i>	—	—	.10	—	.16	—	—	—	—	.04	—	.16	—	—	.77
<i>t</i>	—	—	.10	—	.02	—	—	—	—	.04	—	.08	—	—	.35
<i>u</i>	—	—	0.20	—	0.02	—	—	—	—	0.04	—	0.04	—	—	0.53

TABLE VIIc
ASTROMETRIC RESULTS—V TAURI

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Des.	$\bar{x}$ Astrog. 1901.115	$\bar{x}$ R19 1902.148	Aver. Dev. from Mean	$\bar{x}$ R4242 1921.168	Aver. Dev. from Mean	$\bar{x}$ R19 Astrog.	$\bar{x}$ R4242 R19	$\bar{y}$ Astrog.	$\bar{y}$ R19	Aver. Dev. from Mean	$\bar{y}$ R4242	Aver. Dev. from Mean	$\bar{y}$ R19 Astrog.	$\bar{y}$ R4242 R19	P.M. for 19,020 Yr.
1:.....	-1403.50	-1403.54	0.88	-1400.85	0.73	-0.04	+2.69	-1222.22	-1222.71	0.15	-1222.93	0.08	-0.49	-0.22	2.70
2:.....	+1748.95	+1752.35	2.19	+1750.72	.35	+3.40	-1.63	-1817.65	-1820.64	1.38	-1818.74	.16	-2.98	+1.90	2.50
4*:.....	-167.18	-167.64	0.10	-168.12	.24	-0.46	+0.48	+547.99	+548.32	0.19	+548.24	.06	+0.33	-0.08	0.49
5*:.....	+503.26	+504.04	.34	+503.00	.10	+ .78	-1.04	-729.23	-729.72	.42	-729.27	.06	- .49	+ .45	1.13
6*:.....	-624.19	-624.15	.05	-624.25	0.02	+ .04	-0.10	-626.87	-626.92	.04	-627.31	.02	- .05	-0.39	0.40
7:.....		+623.00	.96	+622.04	1.00		- .96		-1268.54	.96	-1267.12	.22		+1.42	1.71
8*:.....	-85.22	-84.89	.04	-84.40	0.07	+ .33	+ .49	+361.08	+361.08	.06	+361.25	.04		+0.17	0.52
10*:.....	+203.35	+203.12	.22	+203.26	.14	- .23	+ .14	+507.55	+507.55	.06	+507.58	.02		+0.03	.15
11*:.....	-294.44	-293.88	.00	-294.34	.01	+ .56	- .46	-297.65	-297.40	.06	-297.78	.01	+ .25	- .38	.60
17*:.....	-542.56	-542.23	.00	-542.45	.06	+ .33	- .22	-494.91	-494.40	.06	-494.09	.05	+ .51	+ .31	.38
18*:.....	-321.98	-322.50	.04	-321.62	.10	- .52	+ .88	-874.23	-874.34	.20	-874.12	.13	- .11	+ .22	.91
21*:.....	-776.32	-776.50	.06	-776.62	.02	- .18	- .12	-871.27	-871.60	.04	-871.12	.38	- .33	+0.48	0.49
22:.....		-247.29	.21	-248.01	.03		- .72		-530.40	.30	-529.38	.02		+1.02	1.25
23*:.....	-144.91	-144.90	.12	-144.56	.04	+0.01	+0.34	+515.22	+514.64	.03	+514.76	.02		+0.12	0.36
24n:.....		+305.86	.14	+304.84	.18		-1.02		+600.50	.18	+600.15	.02		- .35	1.08
24s:.....		+305.92	.10	+304.45	.14		-1.47		+592.52	.28	+592.54	.10		+ .01	1.47
25:.....		+171.47	.18	+170.92	.28		-0.55		+210.26	.17	+210.97	.03		+ .71	.90
27:.....		+410.42	.33	+409.21	.08		-1.21		-106.65	.22	-105.61	.11		+ .04	1.21
30:.....		+152.96	.10	+152.62	.12		-0.34		+594.27	.01	+594.97	.08		+ .70	0.78
l:.....		-155.38	.09	-156.32	.10		-0.94		+272.68	.08	+273.20	.12		+ .52	1.07
m=x:.....		+237.24	.00	+236.14	.04		-1.10		+621.92	.04	+622.12	.04		+ .30	1.14
n=q:.....		-2.26	.12	-3.12	.22		-0.86		+516.20	.05	+516.60	.04		+ .40	0.95
o=r:.....		-26.92	.06	-25.63	.01		-1.29		+537.04	.00	+537.48	.22		+ .44	1.30
p=u:.....		+102.39	.12	+100.70	.08		-1.69		+571.63	.08	+571.68	.16		+ .05	1.69
t:.....		+242.44	0.02	+240.44	0.12		-2.00		+442.76	0.22	+492.98	0.04		+0.22	2.01

TABLE VIII
ASTROMETRIC RESULTS—R CAMELOPARDALIS

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Des.	Astrog. 1899.459	x R2070 1908.226	Aver. Dev. from Mean	x R4274 1921.275	Aver. Dev. from Mean	x R2070 Astrog.	x R4274 R2070	y Astrog.	y R2070	Aver. Dev. from Mean	y R4274	Aver. Dev. from Mean	y R2070 Astrog.	y R4274 R2070	P.M. for 13.049 Yr.
1*	-300.24	-300.12	0.04	-299.91	0.08	+0.12	+0.21	+431.01	+431.24	0.12	+431.48	0.10	+0.24	+0.24	0.32
4*	+677.91	+677.88	.10	+677.68	.16	-.03	-.20	-424.29	-424.46	.23	-424.30	.08	-.17	+.16	.26
7*	+471.30	+471.21	.06	+471.18	.06	-.09	-.03	+861.81	+862.18	.20	+861.88	.04	+.37	+.30	.30
10*	+523.68	+523.84	.19	+523.86	.00	+.04	+.02	-142.68	-142.49	.01	-142.44	.02	+.19	+.05	.05
11*	-140.73	-140.77	.04	-140.68	.04	-.04	+.09	+287.91	+287.61	.04	+287.23	.10	-.30	-.38	.39
12*	-568.41	-567.98	.00	-567.95	.08	+.43	+.03	+1063.08	+1063.25	.02	+1062.87	.17	+.17	-.38	.38
13*	+349.02	+349.08	.20	+349.23	.07	+.06	+.15	+350.07	+349.76	.16	+349.88	.08	-.31	+.12	.19
14*		+757.64	.04	+757.33	.06		.31		+481.50	.50	+481.39	.12		+.11	.33
16*	-410.52	-411.11	.05	-411.14	.07	-.59	-.03	+731.13	+730.96	.04	+731.46	.06	-.17	+.50	.50
18*	-586.71	-586.76	.12	-585.84	.20	-0.05	+.92	+703.95	+703.94	.11	+703.50	.10	-0.01	+.44	1.02
19*		+564.33	.12	+563.40	.23		.97		+220.58	.14	+220.94	.10		+.36	1.04
<i>l</i> .		+730.60	.24	+729.92	.02		.68		+11.74	.20	+12.08	.04		+.34	.76
<i>m</i> .		+819.16	.20	+818.32	.15		-0.84		+50.86	.10	+51.03	.14		+.17	.86
<i>n</i> .		+699.69	.37	+697.96	.08		-1.73		-103.58	.12	-103.06	.02		+.52	1.81
<i>o</i> .		+621.25	.15	+620.10	.32		-1.15		+7.60	.20	+7.48	.05		-.12	1.16
<i>p</i> .		+534.60	.22	+533.52	.00		-1.08		+492.33	.31	+491.63	.05		+0.70	1.29
<i>q</i> .		+87.82	.04	-88.06	.20		-0.24		+456.37	.48	+455.50	.06		+1.37	1.39
<i>r</i> .		-247.38	.19	-247.18	.06		+.20		+546.34	.64	+545.08	.08		+1.26	1.27
<i>s</i> .		-157.08	.16	-157.52	.04		-.44		-544.57	.62	-543.20	.08		+1.37	1.44
<i>t</i> .		-157.51	.44	-157.54	.14		-.03		+579.46	.53	+578.46	.42		+1.00	1.00
<i>u</i> .		-151.96	0.21	-152.22	0.12		-0.26		+639.12	0.56	+638.18	0.20		+0.94	0.98

TABLE VIIe
ASTROMETRIC RESULTS—R SERPENTIS

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Des.	x Astrog. 1901.469	x R63 1904.464	Aver. Dev. from Mean	x R4264 1921.258	Aver. Dev. from Mean	x R63 Astrog.	x R4264 R63	y Astrog.	y R63	Aver. Dev. from Mean	y R4264	Aver. Dev. from Mean	y R63 Astrog.	y R4264 R63	P.M. for 16,794 Yr.
3: . . .	+1149".22	+1148".74	0".48	+1147".86	0".08	-0".48	-0".88	+370".89	+369".56	0".06	+368".72	0".38	-1".33	-0".84	1".22
4: . . .		+1958.44	.71	+1956.15	.13		-2.29		+1057.94	.53	+1058.76	.94		+ .82	2.43
5: . . .	+1052.83	+1053.01	.12	+1053.44	.12	+ .18	+0.43	-1017.39	-1017.60	.04	-1018.11	.79	-0.21	- .51	0.67
7: . . .	+1165.15	+1165.13	.23	+1164.70	.10	+ .02	- .43	-310.05	-310.24	.27	-309.31	.62	- .19	+ .93	1.02
9: . . .	-1241.95	-1241.84	.04	-1240.86	.12	+ .11	+ .98	-85.62	-86.35	.34	-85.90	.45	- .73	+ .45	1.08
10: . . .	+961.56	+961.70	.18	+961.76	.24	+ .14	+ .06	-618.92	-618.70	.12	-618.48	.66	+ .22	+ .22	0.23
12*: . . .	-112.70	-112.63	.03	-112.18	.28	+ .07	+ .45	-984.98	-985.02	.10	-985.28	.09	- .04	- .26	.52
13*: . . .	+167.97	+168.00	.05	+168.00	.12	+ .03	.00	-952.96	-953.01	.05	-952.73	.07	- .05	+ .28	.28
15*: . . .	-399.97	-400.16	.14	-400.11	.03	- .19	+ .05	-403.84	-403.96	.05	-403.97	.07	- .12	- .01	.05
16*: . . .	-607.21	-607.23	.06	-607.27	.15	- .02	- .04	-712.87	-712.80	.12	-712.70	.06	+ .07	+ .10	.11
17*: . . .	-544.96	-544.92	.18	-544.82	.12	+ .06	+ .10	-200.11	-199.97	.02	-200.07	.01	+ .14	+ .10	.14
18*: . . .	-625.76	-625.71	.03	-625.78	.02	+0.05	- .07	+426.44	+426.38	.05	+426.43	.01	-0.06	+ .05	.09
20: . . .		-708.22	.04	-708.74	.14		- .52		-222.40	.14	+221.96	.04		- .44	.68
21: . . .		-522.64	.03	-523.08	.12		+ .44		+360.58	.01	+360.46	.04		- .12	.46
<i>l</i> : . . .		-758.06	.04	-758.48	.18				-496.96	.02	-497.04	.02		- .08	.43
<i>m</i> = <i>r</i> : . . .		-662.00	.08	-661.73	.10		+ .27		-565.32	.03	-565.16	.10		+ .16	.31
<i>n</i> : . . .		-586.04	.04	-585.62	.28		+ .42		-257.40	.01	-257.81	.07		+ .41	.59
<i>o</i> : . . .		-819.72	.04	-820.40	.13		- .68		+120.85	.17	+121.04	.04		+ .19	.71
<i>p</i> : . . .		-554.34	.04	-554.87	.03		- .53		+346.10	.06	+346.60	.02		+ .50	.73
<i>q</i> : . . .		-567.19	.13	-567.28	.24		- .08		-630.40	.14	-630.86	.17		- .46	.47
<i>s</i> : . . .		-588.86	.06	-589.30	.10		- .44		-579.28	.30	-579.63	.10		- .35	.56
<i>t</i> : . . .		-536.65	.02	-537.09	0.48		-0.44		-586.68	.14	-587.18	0.05		-0.50	0.67
<i>u</i> : . . .		-425.48	0.08						-489.14	0.14					

TABLE VII.
ASTROMETRIC RESULTS—R DRACONIS

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Des.	α Astrog. 1895.421	α R59 1904.445	Aver. Dev. from Mean	α R4291 1921.349	Aver. Dev. from Mean	α R59 Astrog.	α R4291 R59	α Astrog.	α R59	Aver. Dev. from Mean	α R4291	Aver. Dev. from Mean	α R59 Astrog.	α R4291 R59	P.M. for 1890.4 Yr.
1:	-1501.41	-1502.49	0.10	-1501.48	0.19	-1.08	+1.01	+1074.51	+1073.86	0.02	+1076.94	1.87	-0.65	+3.08	3.25
2:	-2556.30	-2555.20	.22	-2553.86	.46	+1.10	+1.34	+1213.86	+1212.81	.18	+1218.64	1.98	-1.05	+5.83	5.97
4:	+304.53	+304.61	.10	+305.07	.11	+0.08	+0.46	-76.08	-75.85	.06	-75.76	0.14	+0.23	+0.09	0.47
5:		+1800.23	.52	+1802.66	.30		+2.43		+1222.25	.58	+1222.28	.46		+0.03	2.43
6:		-469.00	.42	-471.32	.08		-2.32		-1809.43	.08	-1810.42	.44		-0.99	2.52
7:		+534.15	.20	+532.48	.04		-1.67		-1624.72	.12	-1626.66	.38		-1.94	2.56
9:		-993.45	.08	-992.81	.10		+0.64		+165.66	.06	+166.94	.36		+1.28	1.43
12:	-474.45	-474.37	.04	-474.56	.04	+0.08	.19	-121.02	-121.02	.02	-121.17	.02		-0.15	0.24
13:		+1384.92	.03	+1385.68	.26		+0.76		+328.49	.25	+327.98	.15		-0.51	0.91
16:		-342.71	.14	-342.48	.08		.23		+458.26	.28	+459.80	.93		+1.54	1.56
18:	+169.14	+169.62	.08	+169.02	.20	+0.48	.60	-568.53	-568.40	.04	-569.12	.20	+0.13	-0.72	0.94
19:	-18.57	-18.83	.04	-18.76	.12	-0.26	.07	-611.85	-611.51	.05	-611.01	.03	+0.34	+0.50	0.50
20:		-138.98	.05	-138.82	.09	-0.11	.16	-670.05	-670.34	.06	-670.34	.12	-0.46	+0.17	0.23
21:	-827.25	-827.04	.54	-826.33	.14	+0.21	.71	-660.30	-659.39	.18	-657.54	.14	-0.91	+1.85	1.98
22:	+636.12	+635.96	.05	+635.75	.07	-0.16	.21	+47.13	+46.91	.04	+47.00	.14	-0.22	+0.09	0.23
29:	-732.60	-731.35	.54	-731.58	.18	+1.25	.23	-746.91	-745.68	.23	-744.36	.21	+1.23	+1.32	1.34
<i>l</i> :		+183.22	.16	+183.24	.03		.02		-369.93	.04	-369.06	.08		+0.87	0.87
<i>m</i> :		-238.47	.35	-238.86	.00		.39		-815.10	.22	-814.90	.30		+0.20	.44
<i>n</i> :		-152.72	.09	-152.76	.02		-0.04		-523.01	.24	-522.74	.12		+0.27	.27
<i>o</i> :		-120.33	.33	-120.19	.28		.14		-688.10	.02	-687.58	.21		+0.52	.54
<i>p</i> :		-48.99	.24	-49.50	.02		.51		-479.46	.09	-479.30	.06		+0.16	.53
<i>q</i> :		+659.73	.16	+659.92	.05		.19		+248.72	.10	+248.94	.00		+0.22	.29
<i>r</i> :		+610.86	.03	+611.21	.01		.35		+277.08	.02	+276.92	.14		-0.16	.38
<i>s</i> :		+542.04	.08	+542.16	.10		.12		+136.09	.16	+135.76	.18		-0.33	.35
<i>t</i> :		+552.93	.02	+553.10	.08		.83		-54.28	.26	-54.65	.03		-0.37	.91
<i>u</i> :		+463.50	0.05	+463.06	0.04		-0.44		-234.43	0.28	-234.48	0.18		-0.05	0.44

TABLE VII_g
ASTROMETRIC RESULTS—R VULPECULAE

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Des.	σ Astrog. 1894.700	τ R130 1904.831	Aver. Dev. from Mean	σ R4021 4034 4204 1920.175	Aver. Dev. from Mean	τ R130 — Astrog.	R4021 4034 4204 —R130	σ Astrog.	σ R130	Aver. Dev. from Mean	σ R4021 4034 4204	Aver. Dev. from Mean	σ R130 — Astrog.	R4021 4034 4204 —R130	P.M. for 15344 Yr.
1:	-1677.11	-1678.06	0.36	-1678.14	0.75	-0.95	-0.08	-1774.26	-1775.56	0.42	-1773.59	0.72	-1.30	+1.97	1.97
2:	-717.50	-719.01	.29	-717.92	.15	-1.51	+1.09	+635.57	+636.10	.30	+635.71	.22	+0.53	-0.39	1.16
3:	+709.42	+709.69	.07	+710.05	.36	-0.27	+0.36	-801.14	-801.24	.04	-801.10	.05	-1.10	+0.14	0.38
5:	+581.29	+584.91	.01	+585.28	.17	+1.62	+0.37	-1206.43	-1206.34	.26	-1206.47	.15	+0.09	+0.13	.39
6:	-496.13	-496.60	.02	-496.60	.07	-0.47	.00	-835.42	-835.71	.01	-835.56	.17	-0.29	+0.15	.15
7:	-380.05	-380.04	.12	-379.40	.17	+0.01	.64	-1437.37	-1437.70	.40	-1436.94	.28	-0.33	+0.76	.99
9:	+225.62	+225.53	.03	+225.65	.14	-0.09	+1.12	-840.68	-840.82	.17	-840.84	.14	-0.14	+0.02	.12
11:	-48.22	-48.00	.12	-47.56	.19	+0.22	+0.44	-636.72	-636.28	.14	-636.57	.10	-0.44	-0.29	.53
19:	+76.30	+75.98	.00	+75.77	.22	-0.32	-0.21	+15.02	+15.40	.16	+15.57	.05	+0.38	+0.17	.27
20:	-535.75	-535.03	.08	-535.21	.18	+0.72	-0.18	-450.59	-450.54	.12	-450.75	.32	+0.05	-0.21	.28
23:	+662.87	+663.28	.02	+663.37	.09	+0.41	.09	+293.68	+293.83	.09	+293.82	.18	+0.15	-0.01	.09
24:	+465.55	+465.20	.04	+465.72	.08	-0.35	.52	+308.15	+308.17	.02	+308.13	.22	+0.01	-0.04	.52
25:	-250.43	-250.66	.10	-250.66	.05	-0.23	.00	+250.93	+250.96	.01	+251.17	.08	+0.03	+0.21	.21
26:	+851.08	+851.16	.04	+850.96	.27	+0.08	-0.20	+242.32	+242.11	.05	+241.84	.19	-0.21	-0.27	.34
29:		-209.04	.12	-209.21	.20		-0.17		-416.97	.09	-416.83	.12		+0.14	.22
30:	+740.19	+739.90	.10	+739.53	.28	-0.29	.37	-839.97	-839.84	.00	-839.51	.27	+0.13	+0.33	.49
57:	+91.02	+91.06	.06	+90.74	.11	+0.02	.32	+131.25	+130.78	.12	+130.72	.06	-0.47	-0.06	.33
<i>l</i>		+240.28	.04	+239.86	.45		-0.42		-210.47	.21	-211.03	.13		-0.56	.70
<i>m</i>		+328.48	.14	+327.71	.47		-0.77		-235.83	.16	-235.82	.18		+0.01	.77
<i>n</i>		+525.56	.08	+524.76	.52		-0.80		-296.40	.04	-296.71	.09		-0.31	.86
<i>o</i> = <i>u</i>		+439.42	.06	+438.15	.39		-1.27		-216.77	.04	-217.24	.14		-0.47	.35
<i>p</i> = <i>r</i>		+344.56	.02	+343.74	.30		-0.82		+30.27	.11	+29.68	.16		-0.59	1.01
<i>q</i>		+298.00	.02	+297.37	.35		-0.63		+77.82	.54	+77.05	.22		-0.77	.99
8		+388.86	.00	+387.50	.47		-1.36		-22.09	.15	-22.72	.12		-0.63	1.50
<i>t</i>		+420.86	0.11	+419.83	0.73		-1.03		-168.67	0.19	-169.10	0.32		-0.43	1.11

TABLE VIII
Astronomic Results. R. PARSONS

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Dec.	Astr. 1897.827	$\frac{p}{R10}$ 1901.994	Aver. Dev. from Mean	$\frac{p}{R4298}$ 1921.013	Aver. Dev. from Mean	$\frac{p}{R10}$ Astr.	$\frac{p}{R4298}$ R10	$\frac{p}{R10}$ Astr.	$\frac{p}{R10}$	Aver. Dev. from Mean	$\frac{p}{R4298}$	Aver. Dev. from Mean	$\frac{p}{R10}$ Astr.	$\frac{p}{R4298}$ R10	P.M. for 1902.1 Yr.
1	- 10847.42	- 10857.54	0.09	- 10847.70	0.30	- 17.12	+ 0.84	- 340.98	- 341.54	0.09	- 341.77	0.10	- 0.36	- 0.23	0.87
2	+ 1408.37	+ 1408.48	.70	+ 1404.85	.43	+ 0.11	- 3.63	+ 122.71	+ 122.04	.12	+ 122.04	.04	- .67	8.65	3.65
4	- 1502.69	- 1503.34	.61	- 1501.31	.44	- .65	+ 1.53	- 981.76	- 982.06	.32	- 980.64	.16	- .30	+ 1.42	2.32
9*	+ 258.30	+ 258.73	.34	+ 258.14	.24	+ .43	- 0.59	- 500.62	- 500.43	.43	- 500.94	.07	+ .19	- 0.51	0.78
13		- 162.74	.02	- 162.12	.21		+ .62		+ 303.29	.15	+ 303.42	.10		+ .13	.65
15*	- 647.81	- 618.66	.01	- 648.06	.05	- .26	+ .01	- 689.59	- 689.38	.04	- 689.20	.06		+ .45	.15
16*	+ 625.60	+ 625.46	.01	+ 625.38	.06	- .14	- .08	+ 128.60	+ 128.66	.12	+ 128.69	.01	+ .06	+ .05	.05
17*	- 465.07	- 465.16	.04	- 465.03	.15	- .10	- 0.13	- 289.96	- 289.96	.01	- 290.09	.12		- .13	.15
19		133.92	.06	+ 135.90	.12		+ 1.98		+ 845.23	.04	+ 845.08	.05		- .15	1.98
20		626.90	.18	+ 627.96	.18		+ 1.06		- 36.82	.10	- 36.24	.02		+ .58	1.21
21*	+ 303.25	303.42	.04	+ 303.28	.14	+ .17	- 0.14	- 280.22	- 280.16	.27	- 280.02	.08		+ .14	0.29
23		795.02	.08	+ 796.92	.17		+ 1.90		+ 697.90	.03	+ 698.36	.14		+ .46	1.43
25		60.30	.07	+ 60.28	.07		- 0.02		- 272.21	.22	- 271.74	.00		+ .47	0.17
30*	- 685.00	- 684.76	.02	- 684.48	.24	+ 0.24	+ .28	- 380.74	- 380.90	.00	- 380.98	.04	- 0.16	- .08	.29
l		572.90	.14	+ 572.91	.10		+ .01		+ 120.37	.01	+ 120.92	.02		+ .55	.55
m		550.90	.06	+ 551.24	.16		+ 0.34		+ 271.41	.04	+ 272.01	.07		+ .60	0.69
n		474.53	.09	+ 475.66	.10		+ 1.13		+ 283.74	.06	+ 284.29	.00		+ .55	1.26
o		139.50	.04	+ 140.34	.19		+ 0.84		+ 239.20	.10	+ 239.86	.13		+ .66	1.07
p		727.22	.11	+ 726.54	.23		- .68		- 128.08	.06	- 127.65	.15		+ .43	0.81
r		477.42	.02	+ 477.33	.03		- 0.09		- 90.86	.14	- 90.53	.04		+ .33	0.31
s		429.37	.19	+ 430.63	.01		+ 1.26		- 219.04	.15	- 219.42	.14		- .38	1.32
t		385.60	.06	+ 384.72	.11		- 0.88		- 97.26	.06	- 96.97	.15		+ 0.71	1.16
u		395.74	.06	+ 395.50	.12		- 0.24		- 57.18	.04	- 55.96	.09		+ 1.22	1.25
v		223.00	0.02	+ 224.12	0.04		+ 1.12		+ 63.56	0.04	+ 64.09	0.19		+ 0.53	1.21

PART II. PHOTOMETRIC INVESTIGATION

Table VIII gives a journal of reflector plates taken for the determination of magnitudes of selected stars in the eight fields investigated. The field, plate number, date, type of plate, grating, aperture, exposure time, scale used in measuring, quality of the plate and filter used are given in the ten columns.

For determining photographic magnitudes ordinary plates (Seed 30) are used, and for the photovisual values, Cramer Instantaneous Isochromatic plates with a yellow color filter (J1). This combination had been shown to have a spectral intensity curve similar to that of the human eye.

The photometric scale of the plate is determined by the use of a partial parallel-wire grating, P7b, placed over the open end of the telescope. The grating is composed of twenty-one parallel rods each 3.93 mm in diameter, and separated by free spaces 3.87 mm wide. The constants of this grating, the difference between the central and first diffraction image on either side of the central image, are 2.81 and 3.33 mag. for apertures of 12 and 18 inches, respectively.

Exposures with 12-inch aperture were used in determining the magnitudes of the brighter stars, especially those which were more than 15' from the variable star on the axis. With an aperture of 17½ inches, only stars within 20' were usually sufficiently round for accurate measurement. In each field both 12- and 17½-inch negatives were taken on photographic and photovisual plates. The 17½-inch photographic exposures were made long enough, if possible, to give images of the sixteenth-magnitude comparison stars.

The plates were measured in the Hartmann microphotometer¹ with scales suitable for the two types of plates. Corrections due to the distance of the stars from the optical axis were applied, and the scale of the plate, or the ratio between difference in magnitude and difference in scale reading, $\frac{\Delta m}{\Delta s}$, was determined, from the constants of the grating. This quantity, multiplied by the measured scale value, reduced it to relative magnitude. In some cases a single factor for the length of the scale was not found to be satisfactory, and corrections read from a "Holman"² curve were used to reduce scale values to relative magnitudes.

To change these relative values into a known system of magnitudes required the determination of a zero point. For the photographic magnitudes, this was found by making free exposures of the same length on the same plate of the field investigated and a nearby Harvard standard field; usually an exposure on the Rumford field was made, then one on the standard field, and finally another on the Rumford field. In this way, when both regions were measured in the Hartmann machine, a direct relation could be shown by a curve between scale reading and magnitude of the stars in the standard region. From this curve magnitudes of a few stars in the Rumford field can be determined, which are sufficient to establish a zero point for each grating plate.

For the photovisual magnitudes, zero points were determined by comparison with Kapteyn fields, using the photovisual magnitudes of stars determined by Professor Parkhurst.

Since the two fields photographed on the same plate did not usually have the same zenith distance, corrections due to the differences in the absorption of light by the atmosphere were applied. For the photographic plates, these corrections were obtained from a table made by Wirtz,³ and for the photovisual plates, from a table by Müller.⁴ These corrections were small in every case, varying from 0.00 to 0.07 mag.

In Table IX the photometric results are given. Column 1 gives the designation of the star; column 2, its photographic magnitude; 3, the number of plates upon which each star was measured;

¹ *Astrophysical Journal*, 10, 321, 1899; *ibid.*, 36, 171, 1912.

² *American Journal of Science*, 132, 278, 1882.

³ *Astronomische Nachrichten*, 154, 361, 1901.

⁴ *Die Photometrie der Gestirne*, p. 135, 1897.

TABLE VIII
JOURNAL OF PHOTOMETRIC PLATES

Field	Plate No. Ser. R	Date	Kind of Plate	Grating	Aper. in Inches	Exp. Time	Scale for Meas.	Quality	Color Filter
S Cassiopeiae	{4020	yr. mo. da.							
	{4020	19 9 25	Photog.	P7b	12	30 ^m	R16	fair	
	{4172	19 9 25	Photog.	P7b	12	10	R16		
	{4172	20 10 3	Photog.	P7b	17 $\frac{1}{2}$	40	R16	good	
	{4172	20 10 3	Photog.	P7b	17 $\frac{1}{2}$	10	R16		
	{4178	20 10 6	Photog.	P7b	17 $\frac{1}{2}$	10	R16	fair	
	{4178	20 10 6	Photog.	P7b	17 $\frac{1}{2}$	60	R16		
	{4180	20 10 7	Photog.	P7b	17 $\frac{1}{2}$	75	R16	good	
	{4180	20 10 7	Photog.	P7b	17 $\frac{1}{2}$	10	R16		
	{4183	20 10 8	Photog.	P7b	12	30	R16	good	
	{4183	20 10 8	Photog.	P7b	12	10	R16		
	{4189	20 10 21	Photovis.	P7b	17 $\frac{1}{2}$	40	R5	very good	J ₁
	{4189	20 10 21	Photovis.	P7b	17 $\frac{1}{2}$	10	R5		
	{4211	21 1 8	Photog.	Free	12	15	R16	good	
	{4211	21 1 8	Photog.	Free	12	15	R16		
H.C.O.A ₁	{4211	21 1 8	Photog.	Free	12	15	R16		
S Cassiopeiae	{4220	21 1 22	Photovis.	P7b	12	60	R5	poor	J ₁
Kap. 20	{4233	21 2 17	Photovis.	Free	12	20	R5	good	J ₁
S Cassiopeiae	{4233	21 2 17	Photovis.	Free	12	20	R5		
	4304	21 6 5	Photovis.	P7b	17 $\frac{1}{2}$	60	R5	good	J ₁
	4305	21 6 5	Photovis.	P7b	12	30	R5	good	
U Arietis	{4213	21 1 9	Photog.	Free	12	15	R16	good	
H.C.O.C ₂	{4213	21 1 9	Photog.	Free	12	15	R16	good	
U Arietis	{4213	21 1 9	Photog.	Free	12	15	R16		
	{4223	21 1 23	Photovis.	Free	12	30	R5	good	J ₁
Kap. 22	{4223	21 1 23	Photovis.	Free	12	30	R5		
U Arietis	{4227	21 1 26	Photovis.	P7b	17 $\frac{1}{2}$	60	R5	good	J ₁
	{4232	21 2 5	Photog.	P7b	17 $\frac{1}{2}$	40	R16	fair	
	{4236	21 2 23	Photog.	P7b	12	30	R16	poor	
	{4236	21 2 23	Photog.	P7b	12	5	R16		
	{4237	21 2 23	Photovis.	P7b	12	60	R5	fair	J ₁
	{4243	21 3 3	Photog.	P7b	17 $\frac{1}{2}$	60	R16	poor	
	{4244	21 3 9	Photog.	P7b	12	30	R16	fair	
V Tauri	{4088	20 2 18	Photog.	P7b	12	20	R16	fair	
	{4088	20 2 18	Photog.	P7b	12	3	R16		
	{4231	21 2 4	Photog.	P7b	17 $\frac{1}{2}$	25	R16	good	
	{4235	21 2 19	Photovis.	P7b	17 $\frac{1}{2}$	60	R5	fair	
	{4238	21 2 23	Photovis.	P7b	12	30	R5	fair	J ₁
	{4239	21 2 28	Photog.	P7b	12	30	R16	good	
	{4239	21 2 28	Photog.	P7b	12	2	R16		
	{4241	21 3 2	Photog.	P7b	17 $\frac{1}{2}$	60	R16	good	
	{4241	21 3 2	Photog.	P7b	17 $\frac{1}{2}$	2	R16	good	
V Tauri	{4249	21 3 10	Photog.	Free	12	15	R16	f-g	
H.C.O.C ₃	{4249	21 3 10	Photog.	Free	12	15	R16		
V Tauri	{4249	21 3 10	Photog.	Free	12	15	R16		
	{4270	21 4 10	Photovis.	Free	12	30	R5	good	
Kap. 24	{4270	21 4 10	Photovis.	Free	12	30	R5		J ₁
W Leonis	{4258	21 3 31	Photog.	P7b	17 $\frac{1}{2}$	5	R16	good	
	{4258	21 3 31	Photog.	P7b	17 $\frac{1}{2}$	60	R16		
	{4259	21 3 31	Photog.	P7b	12	30	R16	fair	
	{4262	21 4 2	Photog.	P7b	17 $\frac{1}{2}$	90	R16	doubtful	
	{4262	21 4 2	Photog.	Free	12	15	R16		
	{4263	21 4 2	Photog.	Free	12	15	R16	fair	
H.C.O.C ₆	{4263	21 4 2	Photog.	Free	12	15	R16	fair	
W Leonis	{4266	21 4 7	Photog.	P7b	17 $\frac{1}{2}$	90	R16	fair	
	{4281	21 5 3	Photog.	P7b	17 $\frac{1}{2}$	90	R16	good	
	{4282	21 5 3	Photog.	Free	12	15	R16	good	
H.C.O.C ₅	{4282	21 5 3	Photog.	Free	12	15	R16		
W Leonis	{4282	21 5 3	Photog.	Free	12	15	R16		
	{4285	21 5 5	Photovis.	Free	12	20	R5	fair	J ₁
Kap. 30	{4285	21 5 5	Photovis.	Free	12	20	R5		
W Leonis	{4285	21 5 5	Photovis.	Free	12	20	R5		
	{4289	21 5 7	Photovis.	P7b	17 $\frac{1}{2}$	60	R5	fair	J ₁
	{4292	21 5 8	Photovis.	P7b	12	60	R5	good	J ₁

TABLE VIII—Continued

Field	Plate No. Ser. R.	Date	Kind of Plate	Grating	Aper. in Inches	Exp. Time	Scale for Meas.	Quality	Color Filter
R Camelop.	4267	21 4 8	Photog.	P7b	12	30	R16	good	
	4275	21 4 10	Photog.	P7b	17½	45	R16	good	
H.C.O.A ₂	4276	21 4 10	Photog.	Free	12	15	R16	f-p	
R Camelop.	4276	21 4 10	Photog.	Free	12	15	R16		
	4286	21 5 5	Photog.	P7b	17½	75	R16	fair	
	4293	21 5 8	Photog.	P7b	17½	60	R16	fair	
	4295	21 5 16	Photovis.	P7b	17½	60	R5	good	J ₁
	4296	21 5 16	Photovis.	P7b	12	60	R5	good	J ₁
	4302	21 5 29	Photovis.	Free	12	20	R5	good	J ₁
Kap. 33	4302	21 5 29	Photovis.	Free	12	20	R5		
R Camelop.	4302	21 5 29	Photovis.	Free	12	20	R5		
R Serpentis	4279	21 4 29	Photog.	P7b	17½	62	R16	good	
	4280	21 4 29	Photog.	P7b	12	25	R16	fair	
	4283	21 5 5	Photog.	Free	12	15	R16	f-g	
H.C.O.C ₃	4283	21 5 5	Photog.	Free	12	15	R16		
R Serpentis	4283	21 5 5	Photog.	Free	12	15	R16		
	4300	21 5 28	Photovis.	P7b	17½	60	R5	fair	J ₁
	4303	21 6 5	Photovis.	Free	12	20	R5	good	J ₁
Kap. 35	4303	21 6 5	Photovis.	Free	12	20	R5		
R Serpentis	4303	21 6 5	Photovis.	Free	12	20	R5		
	4307	21 6 11	Photovis.	P7b	12	60	R5	good	J ₁
R Draconis	4287	21 5 5	Photog.	P7b	12	30	R16	good	
	4290	21 5 7	Photog.	P7b	17½	85	R16	fair	
	4294	21 5 8	Photog.	Free	12	15	R16	excellent	
H.C.O.A ₂	4294	21 5 8	Photog.	Free	12	15	R16		
R Draconis	4294	21 5 8	Photog.	Free	12	15	R16		
	4298	21 5 28	Photovis.	P7b	17½	60	R5	good	J ₁
	4299	21 5 28	Photovis.	Free	12	20	R5	fair	
Kap. 36	4299	21 5 28	Photovis.	Free	12	20	R5		
R Draconis	4299	21 5 28	Photovis.	Free	12	20	R5		
	4301	21 5 29	Photovis.	P7b	12	60	R5	good	J ₁
R Vulpeculae	4019	19 9 25	Photog.	P7b	12	30	R16	fair	
	4019	19 9 25	Photog.	P7b	12	10	R16	fair	
	4170	20 10 1	Photog.	P7b	17½	40	R16	fair	
	4170	20 10 1	Photog.	P7b	17½	10	R16		
	4171	20 10 3	Photog.	P7b	17½	40	R16	good	
	4171	20 10 3	Photog.	P7b	17½	10	R16		
	4176	20 10 6	Photog.	P7b	12	5	R16	fair	
	4176	20 10 6	Photog.	P7b	12	30	R16		
H.C.O.C ₁₁	4179	20 10 7	Photog.	Free	12	20	R16	fair	
R Vulpeculae	4179	20 10 7	Photog.	Free	12	20	R16		
H.C.O.C ₁₁	4179	20 10 7	Photog.	Free	12	20	R16		
H.C.O.C ₁₁	4181	20 10 8	Photog.	Free	12	20	R16	fair	
V Vulpeculae	4181	20 10 8	Photog.	Free	12	20	R16		
H.C.O.C ₁₁	4181	20 10 8	Photog.	Free	12	20	R16		
R Vulpeculae	4184	20 10 9	Photog.	P7b	12	30	R16	good	
	4184	20 10 9	Photog.	P7b	12	10	R16		
	4190	20 10 29	Photovis.	P7b	17½	55	R5	fair	J ₁
	4190	20 10 29	Photovis.	P7b	17½	8	R5		
Kap. 40	4200	20 11 16	Photovis.	Free	12	60	R5	fair	J ₁
R Vulpeculae	4200	20 11 16	Photovis.	Free	12	60	R5		
Kap. 40	4200	20 11 16	Photovis.	Free	12	60	R5		
R Vulpeculae	4201	20 11 18	Photovis.	P7b	17½	90	R5	fair	J ₁
	4205	20 12 8	Photovis.	P7b	12	60	R5	good	J ₁
	4212	21 1 9	Photovis.	Free	12	30	R5	fair	J ₁
Kap. 40	4212	21 1 9	Photovis.	Free	12	30	R5	fair	
R Pegasi	4207	21 1 4	Photog.	Free	12	10	R16	fair	
H.C.O.C ₁₂	4207	21 1 4	Photog.	Free	12	10	R16		
R Pegasi	4210	21 1 8	Photog.	Free	12	10	R16	f-g	
H.C.O.C ₁₂	4210	21 1 8	Photog.	Free	12	10	R16	f-g	
R Pegasi	4210	21 1 8	Photog.	Free	12	10	R16		
	4215	21 1 12	Photog.	P7b	17½	60	R16	fair	
	4215	21 1 12	Photog.	P7b	17½	5	R16	fair	
	4217	21 1 22	Photovis.	P7b	17½	50	R5	poor	J ₁
	4221	21 1 23	Photovis.	Free	12	20	R5	f-g	J ₁
Kap. 42	4221	21 1 23	Photovis.	Free	12	20	R5		
R Pegasi	4224	21 1 26	Photovis.	P7b	17½	55	R5	good	J ₁

4, the average deviation of the different determinations from the mean value; 5, the photovisual magnitude; 6 and 7, columns for photovisual magnitudes corresponding to 3 and 4; and 8, the color index. Columns 9 and 10 give a comparison with Rumford visual magnitudes as determined by Professor Parkhurst. Column 9 gives the magnitude, and column 10, the difference between the Rumford visual and my photovisual magnitude. The color indices for the q - u stars in column 8 are determined by comparing my photographic magnitudes with these Rumford visual magnitudes.

TABLE IX_a
PHOTOMETRIC RESULTS—S CASSIOPEIAE

1	2	3	4	5	6	7	8	9	10
DES.	PHOTOG. MAG	NO. OF PLATES	AVER. DEV. FROM MEAN	PV. MAG.	NO. OF PLATES	AVER. DEV. FROM MEAN	COLOR INDEX	COMP. WITH RUMFORD VISUAL	
								R. Vis.	R-Pv.
1.....	8 ^m 24	4	0 ^m 20	6 ^m 94	4	0 ^m 07	+1 ^m 30	7.11	+0.17
2.....	8.01	3	.14	6.92	3	.21	1.09	7.32	+ .40
3.....	7.99	4	.16	6.84	4	.12	1.15	7.02	+ .18
4.....	8.84	5	.12	8.41	4	.07	0.43		
5.....	9.05	1		8.86	2	.00	.19	8.85	— .01
9.....	10.24	3	.33	9.79	5	.12	.45	9.92	+ .13
10.....	10.49	6	.07	9.87	5	.12	.62	10.07	+ .20
11.....	10.49	6	.10	10.20	5	.08	.29	10.16	— .04
12.....	10.49	6	.06	10.34	5	.13	.15	10.30	— .04
14.....	10.71	6	.04	10.34	5	.11	0.37		
A 20.....	12.85	6	.11	11.01	4	.09	1.84	10.69	— .32
B 22.....	12.07	6	.08	11.31	4	.08	0.76	10.93	— .38
C 25.....	12.09	6	.14	11.43	3	.03	.66	10.96	— .47
D 27.....	12.08	6	.10	11.80	2	0.22	.28	11.15	— .65
E 35.....	12.49	6	.12	12.41	1		.08	11.65	—0.76
<i>l</i>	15.21	3	.12						
<i>m</i>	15.52	3	.16						
<i>n</i>	15.28	3	.15						
<i>o</i>	15.74	2	.11						
<i>p</i>	15.25	3	0.20						
<i>q</i>	15.99	1					1.74	14.25	
<i>r</i>	16.04	1					1.68	14.36	
<i>s</i>	16.87	1					2.38	14.49	
<i>t</i>	16.13	1					1.78	14.35	
<i>u</i>	16.44	1					+1.88	14.56	

TABLE IXb

PHOTOMETRIC RESULTS—U ARIETIS

1	2	3	4	5	6	7	8	9	10
DES.	PHOTOG. MAG.	NO. OF PLATES	AVER. DEV. FROM MEAN	PV. MAG.	NO. OF PLATES	AVER. DEV. FROM MEAN	COLOR INDEX	COMP. WITH RUMFORD VISUAL	
								R. Vis.	R-Pv.
3:.....	9 ^m 60	2	0 ^m 08	8 ^m 44	2	0 ^m 06	+1 ^m 16	8.78	+0.34
4:.....								9.05	
5:.....								9.86	
6:.....	9.51	5	.15	9.40	3	.05	0.11	9.96	.56
9:.....	10.39	5	.09	10.15	3	.13	.24		
10:.....	10.35	5	.08	9.99	3	.05	.36		
12:.....	10.30	4	.13	10.00	3	.05	.30		
13:.....									
14:.....	10.54	4	.10	10.23	3	.11	.31		
16:.....	11.75	5	.08	11.19	2	0.26	.56		
A 17:.....	12.37	5	.15	11.55	1		.82	11.62	.07
B 19:.....	12.53	4	.12	12.05	1		.48	12.11	+0.06
20:.....	12.32	4	.07	11.81	1		.51		
C 21:.....	12.84	2	.22					12.40	
D 22:.....	12.48	3	.10					12.26	
E 23:.....	12.64	4	.08					12.42	
25:.....	12.79	3	.26	12.27	1		0.52		
26:.....	13.40	2	0.12						
l.....	14.15								
m.....	14.73								
n.....	14.98								
o.....									
p.....									
q.....	15.34						1.50	13.84	
r.....									
s.....									
t.....	14.90						+0.95	13.95	
u.....									

TABLE IXc
PHOTOMETRIC RESULTS—V TAURI

1	2	3	4	5	6	7	8	9	10
Des.	PHOTOG. MAG.	NO. OF PLATES	AVER. DEV. FROM MEAN	Pv. MAG.	NO. OF PLATES	AVER. DEV. FROM MEAN	COLOR INDEX	COMP. WITH RUMFORD VISUAL	
								R. Vis.	R-Pv.
1:.....	7 ^m 88	1		6 ^m 00			+1 ^m 88	7.04	+1.04
2:.....								7.21	
4:.....	9.22	5	0 ^m 13	8.71	3	0 ^m 02	0.51		
5:.....	9.36	5	.22	8.63	3	.18	0.73	9.03	+0.40
6:.....	10.40	5	.07	9.22	3	.18	1.18	9.40	+ .18
7:.....	10.23	3	.22	9.25	2	.08	+0.98		
8:.....	10.47	5	.10	9.75	3	.20	+ .72	9.96	+ .21
10:.....	10.14	5	.10	9.83	3	.03	+0.31	10.10	+ .27
11:.....	10.82	5	.13	9.51	3	0.16	+1.31	9.91	+ .40
17:.....	11.56	5	.05	10.37	1		+1.18		
18:.....	11.87	5	.09	10.70	1		+1.17		
21:.....	11.60	5	.12	10.95	1		+0.65		
A 22:.....	12.20	5	.05	11.59	1		+ .61	11.58	— .01
23:.....	11.81	5	.10	12.14	1		— .33		
B 24n.....									
24s.....									
C 25:.....	12.19	4	.24					11.57	
D 27:.....	12.64	4	.16					11.54	
E 30:.....	12.43	5	.16	12.27			+0.16	11.73	—0.54
l.....	15.44	2	.26						
m=s.....	15.90	1					+2.38	13.52	
n=q.....	16.01	1					+2.02	13.99	
o=r.....	15.93	1					+2.19	13.74	
p=u.....	16.25	1					+1.87	14.38	
t.....	16.40	1					+2.07	14.33	
3:.....	9.06	2	0.21	8.68			+0.38		

TABLE IX*d*
 PHOTOMETRIC RESULTS—R CAMELOPARDALIS

1	2	3	4	5	6	7	8	9	10
DES.	PHOTOG. MAG.	NO. OF PLATES	AVER. DEV. FROM MEAN	PV. MAG.	NO. OF PLATES	AVER. DEV. FROM MEAN	COLOR INDEX	COMP. WITH RUMFORD VISUAL	
								R. Vis.	R-Pv.
1.....	9 ^M 73	5	0 ^M 06	8 ^M 44	2	0 ^M 04	+1 ^M 29	8.44	0.00
2.....	9.74	1		9.26	2	.02	0.48		
3.....	9.62	1		9.42	2	.06	.20		
4.....	9.82	5	.09	9.44	3	.05	0.38	9.48	+ .04
7.....	11.22	5	.10	10.11	3	.06	1.11	10.07	- .04
10.....	11.93	5	.19	11.16	2	.02	0.77	10.81	- .35
11.....	11.37	5	.10	11.28	2	.33	.09	10.80	- .18
12.....	11.60	5	.06	11.46	2	0.12	.14		
A 13.....	12.07	5	.06	11.39	1		.68	11.15	- .24
B 14.....	12.48	5	.11	12.19	1		.29	11.62	- .57
C 16.....	13.27	5	.16	12.44	1		.83	11.81	-0.63
D 18.....	13.20	5	.14	13.06	1		.14	12.06	-1.00
E 19.....	13.57	5	.18	12.79	1		0.78	12.25	-0.54
l.....	14.63	3	.17						
m.....	14.78	3	.23						
n.....	16.23	3	.30						
o.....									
p.....	16.85	1							
q.....	16.31	2	.31				2.06	14.25	
r.....	16.42	2	0.28				2.00	14.42	
s.....	16.14	1					+1.14	15.00	
t.....									
u.....									

TABLE IX_c
PHOTOMETRIC RESULTS—R SERPENTIS

1	2	3	4	5	6	7	8	9	10
DES.	PHOTOG. MAG.	NO. OF PLATES	AVER. DEV. FROM MEAN	PV. MAG.	NO. OF PLATES	AVER. DEV. FROM MEAN	COLOR INDEX	COMP. WITH RUMFORD VISUAL	
								R. Vis.	R-Pv
3:.....	6 ^m 75	1		7 ^m 30	2	0 ^m 22	-0 ^m 55	7.25	-0.05
4:.....				7.19	1			7.41	+ .22
5:.....	9.02	2	0 ^m 30	8.67	3	.16	+ .35	8.67	.00
6:.....	8.76	1		9.24	2	.04	- .48		
7:.....	10.07	3	.14	9.44	3	.07	+ .63	9.43	- .01
8:.....	9.73	2	.13	9.85	2	.08	- .12		
9:.....	10.09	2	.06	10.48	3	.10	- .39	10.30	- .18
10:.....	10.41	3	.13	10.26	3	.09	+ .15	10.25	- .01
12:.....	10.87	3	.14	10.36	2	.23	+ .51		
13:.....	10.74	3	.04	10.52	2	0.11	+ .22		
A 15.....	11.25	3	.11	11.38	1		- .13	10.89	- .49
B 16.....	11.05	3	.08	11.14	1		- .09	10.89	- .25
C 17.....	12.00	3	.12					10.98	
18:.....	11.55	3	.04	12.01	1		- .46		
19:.....	12.20	3	.04	11.92	1		+ .28		
D 20.....	12.09	3	.28	12.08	1		+0.01	11.39	-0.69
E 21.....	12.12	3	0.03					11.69	
l.....	14.51	1							
m=r.....	15.39	1					+1.65	13.74	
n.....	15.99	1							
o.....	15.60	1							
p.....	15.79	1							
q.....	14.17	1					+0.97	13.20	
s.....	15.55	1					+1.39	14.16	
t.....	16.43	1					+1.85	14.58	
u.....	16.77	1					+1.87	14.90	

TABLE IX^f
 PHOTOMETRIC RESULTS—R DRACONIS

1	2	3	4	5	6	7	8	9	10
DES.	PHOTOG. MAG.	NO. OF PLATES	AVER. DEV. FROM MEAN	PV. MAG.	NO. OF PLATES	AVER. DEV. FROM MEAN	COLOR INDEX	COMP. WITH RUMFORD VISUAL	
								R. Vis.	R-Pv.
1:.....	7 ^M 75	1		6 ^M 67	1		+1 ^M 08	6.68	+0.01
2:.....								7.38	
4:.....	9.67	3	0 ^M 25	8.35	2	0 ^M 24	1.32	8.43	+ .08
5:.....				8.91	2	.03			
6:.....	9.79	1		8.87	1		0.92		
7:.....	9.08	1		8.84	1		.24		
9:.....	9.69	2	.07	9.53	3	.09	0.16	9.59	+ .06
12:.....	10.77	3	.10	9.67	3	.07	1.10	9.86	+ .19
13:.....	10.46	2	.20	10.11	3	.24	0.35	10.10	— .01
16:.....	11.02	3	.04	10.54	2	.05	.48	10.39	— .15
A 18:.....	11.20	3	.27	10.74	2	.08	.46	10.55	— .19
B 19:.....	11.84	3	.26	11.31	2	0.04	.53	10.95	— .36
C 20:.....	12.07	3	.07	11.97	1		.10	11.36	— .61
D 21:.....	12.54	2	.28					11.61	
E 22:.....	12.49	3	.24	11.90	1		.59	11.48	—0.42
27:.....	12.86	3	.13	12.54	1		.32		
29:.....	13.03	2	0.17						
l.....	14.35	1							
m.....	14.62	1							
n.....	14.51	1							
o.....	14.91	1							
p.....	15.47	1							
q.....	15.10	1					.86	14.24	
r.....	15.51	1					0.90	14.61	
s.....	15.89	1					1.26	14.63	
t.....	15.50	1					1.37	14.13	
u.....	15.05	1					+0.99	14.06	

TABLE IX_g
 PHOTOMETRIC RESULTS—R. VULPECULAE

1	2	3	4	5	6	7	8	9	10
DES.	PHOTOG. MAG.	NO. OF PLATES	AVER. DEV. FROM MEAN	Pv. MAG.	NO. OF PLATES	AVER. DEV. FROM MEAN	COLOR INDEX	COMP. WITH RUMFORD VISUAL	
								R. Vis.	R-Pv.
1:									
2	8 ^M 51	4	0 ^M 11	7 ^M 26	3	0 ^M 34	+1 ^M 25	7.17	-0.09
3	8.70	5	.13	7.44	4	.21	+1.26		
5	9.13	5	.18	8.75	5	.10	+0.38	9.01	+ .26
6	10.44	7	.09	9.44	5	.09	+1.00	9.53	+ .09
7:	10.34	5	.25	9.18	3	.09	+1.16	9.61	+ .43
8	9.72	2	.06	9.65	1		+0.07		
9	10.47	7	.09	9.78	5	.23	+ .69	9.95	+ .17
11	10.53	7	.13	9.81	5	.16	+ .72	10.14	+ .33
19	10.51	7	.09	10.32	4	.12	+ .19		
20	10.51	7	.12	10.34	4	.20	+ .17		
A 23	11.29	7	.17	10.31	4	.22	+ .98	10.48	+ .17
B 24	10.27	7	.14	10.33	4	.13	-0.06	10.32	- .01
C 25	11.27	7	.13	10.17	4	.16	+1.10	10.34	+ .17
D 26									
E 29	11.87	7	.15	10.61	4	.24	+1.26	10.58	-0.03
30	12.08	7	.15	10.88	3	.39	+1.20		
57	12.01	7	.15	11.95	2	0.07	+0.06		
l	14.56	2	.02						
m	15.50	2	.09						
n	14.48	2	.04						
o=u	16.07	2	.06				+1.57	14.50	
p=r	15.43	2	.03				+1.08	14.35	
q	15.63	2	.05				+1.42	14.21	
s	16.18	2	.04				+1.31	14.87	
t	16.24	2	0.12				+1.23	15.01	

TABLE IX^h
PHOTOMETRIC RESULTS—R PEGASI

1	2	3	4	5	6	7	8	9	10
DES.	PHOTOG. MAG.	NO. OF PLATES	AVER. DEV. FROM MEAN	PV. MAG.	NO. OF PLATES	AVER. DEV. FROM MEAN	COLOR INDEX	COMP. WITH RUMFORD VISUAL	
								R. Vis.	R-Pv.
2.....				7 ^m 66	2	0 ^m 34		7.07	-0.59
3.....				8.01	1			8.30	+ .29
4.....				8.24	1				
9.....	10 ^m 96	2	0 ^m 16	9.97	3	.03	+0 ^m 99	9.85	- .12
13.....	10.63	3	.23	11.04	2	.30	- .41	10.41	- .63
15.....	11.99	2	.02	11.32	2	0.15	+ .67	10.48	- .84
16.....	11.97	3	.09	11.42	1		+ .55		
A 17.....	11.98	2	.08	11.75	1		+0.23	10.89	- .86
B 19.....	13.02	2	.04	11.88	1		+1.13	11.44	- .44
C 20.....	12.71	2	.16	11.90	1		+ .81	11.32	- .58
D 21.....	12.32	3	.08	11.67	1		+ .65		
E 23.....	12.98	1		12.05	1		+0.93	11.56	-0.49
25.....	12.88	2	.26	11.79	1		+1.09		
30.....	12.70	2	0.25	12.23	1		+0.47		
l.....	15.04	1							
m.....	13.95	1							
n.....	15.91	1							
o.....	14.67	1							
p.....	15.57	1							
q.....	15.95	1					+1.92	14.03	
r.....	16.07	1					+1.71	14.36	
s.....	16.47	1					+1.60	14.87	
t.....	15.18	1					+1.81	14.37	
u.....	16.26	1					+1.59	14.67	

Table X gives a comparison between photographic magnitudes for stars in two fields, R Camelopardalis and R Pegasi, as determined by Gingrich,¹ and my photographic magnitudes of these same stars. My measures are in almost every case consistently fainter than those of Gingrich, due to a difference in zero point of the two determinations.

Table XI contains an interesting comparison between data given in the *Henry Draper Catalogue of Spectra*² and my results for the fields contained in the portion of the *Catalogue* already

TABLE X
COMPARISON OF GINGRICH AND PARSONS PHOTOGRAPHIC MAGNITUDES

DES.	R CAMPELOPARDALIS			DES.	R PEGASI		
	Ging.	Ps.	Ps-Ging.		Ging.	Ps.	Ps-Ging.
1.....	9 ^m 28	9 ^m 73	+0 ^m 45	9.....	10 ^m 35	10 ^m 96	+0 ^m 61
2.....	8.95	9.74	.79	13.....	10.79	11.30	.51
3.....	9.20	9.62	.42	15.....	11.41	11.99	.58
4.....	9.01	9.82	.81	16.....	11.39	11.97	.58
7.....	10.53	11.22	.69	17.....	11.48	11.98	.50
10.....	11.36	11.93	.57	19.....	12.04	13.02	.98
11.....	10.88	11.37	.49	20.....	12.00	12.71	.71
12.....	11.11	11.60	.49	21.....	11.77	12.32	.55
13.....	11.51	12.07	.56	23.....	12.50	12.98	.48
14.....	11.69	12.48	0.79	25.....	12.12	12.88	.76
16.....	11.93	13.27	1.34	30.....	12.26	12.70	+0.44
19.....	12.90	13.57	+0.67				
Mean.....			+0.67	Mean.....			+0.61

¹ *Astrophysical Journal*, 38, 209, 1913.

² *Annals of Harvard College Observatory*, 91, 92, 1918.

published. The various columns give the following data: Hagen number of star; my photographic magnitude, photovisual magnitude, and color index; the Harvard photographic magnitude, visual magnitude, color index, and spectral type, and the number of the star in the *Harvard Catalogue*. For those stars not too far from the axis to give good images, these two distinct determinations of color indices are in close agreement.

TABLE XI
COMPARISON OF COLOR INDICES AND HARVARD SPECTRAL TYPES

FIELD	STAR	PARSONS			HARVARD				
		Ptg.	Pv.	C.I.	Ptg.	Pv.	C.I.	Spec. Type	Harv. No.
S Cassiopeiae . . .	1	8 ^M 24	6 ^M 94	+1 ^M 30	7 ^M 96	7 ^M 18	+0 ^M 78	G5	7646
	2:	8.01	6.92	1.09	7.16	7.16	.00	A0	8226
	3	7.99	6.84	1.15	7.86	7.08	.78	G5	7655
	4	8.84	8.41	0.43	8.86	8.44	.42	F5	7768
	5:	9.05	8.86	0.19	8.62	8.62	0.00	A0	7176
U Arietis	3:	9.60	8.44	1.16	10.0	8.9	1.1	K2	19697
	6	9.51	9.40	0.11	11.0	10.7	0.3	F	19753
V Tauri	1:	7.88	6.00	1.88	7.72	7.22	.50	F8	30676
	4	9.22	8.71	0.51	9.57	9.07	.50	F8	30843
	5	9.36	8.63	+0.73	9.64	9.08	+0.46	G0	30946

PART III. ASTROMETRIC AND PHOTOMETRIC RELATIONS AND CONCLUSIONS

A study of the relations existing between photographic magnitudes and proper motions in both the x and y directions is given in Tables XII–XIV. Groups of stars in each field were combined according to photographic magnitude (see Table XII). The number of stars included in the means, the mean photographic magnitude, the mean photovisual magnitude, mean color index, and mean proper motions in x and y , both algebraic and arithmetic, are given. Since the intervals between early and late plates vary for different fields, centennial proper motions are tabulated also. The last line of data in each field is for Hagen stars not used as standards.

Combining these means for stars of similar brightness for the eight fields (see Table XIII) and weighting according to the number of stars included in each group, the average centennial proper motions for five different magnitudes are found. Table XIIIa gives similar data for all Hagen stars giving good images not used as standards. In Table XIV the results of the comparison are given. For the first three groups the proper motions are small; for the next group, the fifteenth-magnitude stars, there is a slight increase; and for the sixteenth-magnitude stars the increase is more marked. At first glance this would seem to indicate an increase in proper motions with decreasing brightness, which is contrary to what might be expected.

The last group given in Table XIV contains stars brighter than the fifteenth magnitude which were not used as standards. The average centennial proper motion for this group is 4".59, greater than that of the fifteenth-magnitude stars, and less than that of the sixteenth-magnitude stars. The stars used as standards, included in the first three groups tabulated, would be expected to bring the average lower than for the other groups, since when the plate constants are applied to these stars their positions would be very close to the astrographic positions on both the early and late plates. For comparison with other groups, the last line of data given is the only one which can have any real meaning. The sixteenth-magnitude stars are in most cases very difficult to measure since they are often just at the limit of visibility, especially on the later plates. Any uncertainties in measurement are increased approximately six times in reducing to centennial proper motions. Considering the sixteenth-magnitude stars, the average proper motion in x or y is 3".94. One-sixth of this, or 0".66, would represent the average difference between early and late measure-

ments of the same stars. For the fifteenth-magnitude stars this is reduced to $0''.40$, and for the brighter stars, not standards, it is $0''.54$.

These values are useful since they give an idea of the accuracy which can be obtained in measuring positions of stars of different magnitudes on the reflector plates. Assuming the average proper motions of these groups of stars to be identical, those stars which give small distinct images, for example, a fifteenth-magnitude star with an hour's exposure with 18-inch aperture on a good night, are the best suited for measurement. Any distortion of the images on the plate will not affect these stars, since only the nucleus will show, as a rule. For the brighter stars, distortion of the image is present but often masked so it is difficult to measure them well. For the very faintest stars, there is often considerable doubt as to the exact position.

Considering the most favorable group, that of the fifteenth-magnitude stars, we find an average proper motion of $0''.03$ annually.

TABLE XII
RELATION OF PROPER MOTIONS TO MAGNITUDE

S Cassiopeiae												
DES.	NO. OF STARS	AVER. PTG. MAG.	AVER. PV. MAG.	AVER. COLOR INDEX	AVERAGE PROPER MOTION							
					13.993 Years				100 Years			
					In x Alg.	In x Arith.	In y Alg.	In y Arith.	In x Alg.	In x Arith.	In y Alg.	In y Arith.
1, 3, 4.....	3	8.36	7.40	+0.96	-0.20	0.20	-0.01	0.15	-1.43	1.43	-0.07	1.07
9-14.....	5	10.48	10.10	.38	-.08	.28	-.18	.27	-.57	2.00	-1.29	1.93
20-35.....	4	12.32	11.59	.52	-.07	.19	+.28	.28	-.50	1.36	+2.00	2.00
$l-p$	5	15.40			+.13	.16	-.24	.45	+.93	1.14	-1.72	3.22
$q-u$	5	16.30			+.06	.24	-.70	.70	+.43	1.72	-5.00	5.00
27.....	1	12.08	11.80	+0.28	-0.31	0.31	+0.51	0.51	-2.22	2.22	+3.64	3.64

U Arietis												
DES.	NO. OF STARS	AVER. PTG. MAG.	AVER. PV. MAG.	AVER. COLOR INDEX	AVERAGE PROPER MOTION							
					15.009 Years				100 Years			
					In x Alg.	In x Arith.	In y Alg.	In y Arith.	In x Alg.	In x Arith.	In y Alg.	In y Arith.
6-14.....	6	10.22	9.95	+0.08	+0.03	0.28	+0.03	0.23	+0.20	1.87	+0.20	1.53
16-20.....	5	12.24	11.65	+0.59	.20	.28	-.10	.22	1.33	1.87	-0.67	1.47
21-23.....	3	12.65			.19	.19	-.19	.19	1.26	1.26	-1.26	1.26
p, m, n, q, t	5	14.82			.22	.28	+.04	.18	1.47	1.87	+0.27	1.20
17, 19.....	2	12.45	11.80	+0.65	.36	.36	-.15	.41	2.40	2.40	-1.00	2.73
21, 22, 23.....	3	12.65			+0.23	0.23	-0.26	0.26	+1.53	1.53	-1.73	1.73

TABLE XII—Continued

V Tauri

DES.	NO. OF STARS	AVER. PTG. MAG.	AVER. PV. MAG.	AVER. COLOR INDEX	AVERAGE PROPER MOTION							
					19.020 Years				100 Years			
					In x Alg.	In x Arith.	In y Alg.	In y Arith.	In x Alg.	In x Arith.	In y Alg.	In y Arith.
4, 5.....	2	9.29	8.67	+0.62	-0.28	0.76	+0.28	0.36	-1.47	3.40	+1.47	1.89
6, 8, 10, 11.....	4	10.46	9.58	.63	+ .02	.30	- .14	.24	+0.11	1.58	-0.74	1.26
17, 18, 21, 22, 23....	5	11.81	11.55	.66	+ .03	.46	+ .42	.42	+0.16	2.42	+2.21	2.21
25, 27, 30.....	3	12.42	12.27	+0.16	-0.70	0.70	+ .48	.48	-3.68	3.68	+2.52	2.52
$l-t$	6	15.99			-1.31	1.31	+ .32	.32	-6.89	6.89	+1.68	1.68
22, 25, 27, 30.....	4	12.36			-0.70	0.70	+0.62	0.62	-3.68	3.68	+3.26	3.26

R Camelopardalis

DES.	NO. OF STARS	AVER. PTG. MAG.	AVER. PV. MAG.	AVER. COLOR INDEX	AVERAGE PROPER MOTION							
					13.049 Years				100 Years			
					In x Alg.	In x Arith.	In y Alg.	In y Arith.	In x Alg.	In x Arith.	In y Alg.	In y Arith.
1, 4.....	2	9.77	8.94	+0.83	0.00	0.20	+0.20	0.20	0.00	1.53	+1.53	1.53
7, 10, 11, 12.....	4	11.53	11.00	.53	+ .03	.04	- .25	.28	+ .23	0.31	-1.92	2.15
13, 14, 16, 18, 19....	5	12.92	12.37	.54	- .05	.48	+ .09	.31	-0.38	3.68	+0.69	2.38
l, m	2	14.70			- .76	.76	+0.25	0.25	-5.82	5.82	+1.92	1.92
n, p, q, r, s	5	16.39			- .66	.74	+1.04	1.04	-5.06	5.67	+7.97	7.97
14, 18, 19.....	3	13.08	12.68	+0.40	-0.12	0.73	-0.06	0.30	-0.92	5.59	-0.46	2.30

R Serpentis

DES.	NO. OF STARS	AVER. PTG. MAG.	AVER. PV. MAG.	AVER. COLOR INDEX	AVERAGE PROPER MOTION							
					16.794 Years				100 Years			
					In x Alg.	In x Arith.	In y Alg.	In y Arith.	In x Alg.	In x Arith.	In y Alg.	In y Arith.
12, 10, 13.....	3	10.67	10.38	+0.29	+0.17	0.17	+0.08	0.25	+1.01	1.01	+0.48	1.49
15, 16, 18, 19, 20....	5	11.63	11.71	-0.08	- .09	.15	- .08	.14	-0.54	0.89	- .48	0.83
l, m, q	3	14.69			- .08	.26	- .13	.23	-0.48	1.55	- .77	1.37
n, o, p, s, t	5	15.87			- .33	.50	- .11	.39	-1.96	2.98	- .65	2.32
10, 20, 21.....	3	11.54			-0.02	0.35	-0.11	0.26	-0.12	2.08	-0.65	1.55

R Draconis

DES.	NO. OF STARS	AVER. PTG. MAG.	AVER. PV. MAG.	AVER. COLOR INDEX	AVERAGE PROPER MOTION							
					16.904 Years				100 Years			
					In x Alg.	In x Arith.	In y Alg.	In y Arith.	In x Alg.	In x Arith.	In y Alg.	In y Arith.
12, 18, 19.....	3	11.27	10.57	+0.70	-0.24	0.29	-0.12	0.46	-1.42	1.72	-0.71	2.72
20, 22.....	2	12.28	11.94	+0.34	- .02	.18	+ .13	.13	-0.18	1.06	+0.77	0.77
$l-p$	5	14.77			- .21	.22	+ .40	.40	-1.24	1.30	+2.37	2.37
$q-u$	5	15.41			- .12	.39	-0.14	0.23	-0.71	2.31	-0.83	1.36
9, 16.....	2	10.36	10.04	+0.32	+0.44	0.44	+1.41	1.41	+2.60	2.60	+8.34	8.34

TABLE XII—*Continued*

R Vulpeculae

Des.	No. of Stars	Aver. Ptg. Mag.	Aver. Pv. Mag.	Aver. Color Index	AVERAGE PROPER MOTION							
					15.344 Years				100 Years			
					In x Alg.	In x Arith.	In y Alg.	In y Arith.	In x Alg.	In x Arith.	In y Alg.	In y Arith.
2, 3, 6.....	3	9.19	8.04	+1.15	+0.48	0.48	−0.03	0.23	+3.13	3.13	−0.20	1.50.
9, 11, 19, 20, 23, 24, 25	7	10.67	10.15	+0.52	+ .11	.22	− .03	.14	+0.72	1.43	− .20	0.91
29, 30, 57.....	3	11.97	11.15	+0.82	− .29	.29	+ .14	.18	−1.89	1.89	+0.91	1.17
l, m, n, p	4	14.97			−0.70	0.70	− .36	.37	−4.56	4.56	−2.35	2.41
o, q, s	4	16.01			−1.07	1.07	− .57	.57	−6.97	6.97	−3.71	3.71
2, 29.....	2	10.19	8.94	+1.25	+0.46	0.63	−0.12	0.26	+3.00	4.11	−0.78	1.69

R Pegasi

Des.	No. of Stars	Aver. Ptg. Mag.	Aver. Pv. Mag.	Aver. Color Index	AVERAGE PROPER MOTION							
					19.021 Years				100 Years			
					In x Alg.	In x Arith.	In y Alg.	In y Arith.	In x Alg.	In x Arith.	In y Alg.	In y Arith.
9, 13.....	2	10.79	10.50	+0.26	+0.01	0.60	−0.19	0.32	+0.05	3.15	−1.00	1.68
15–30.....	9	12.51	11.88	+0.73	.54	.62	+ .11	.19	2.84	3.26	+0.58	1.00
$l-p$	5	15.03			.33	.60	+ .56	.56	1.73	3.15	+2.94	2.94
$q-u$	5	16.19			0.25	0.70	+ .48	.63	1.31	3.68	+2.52	3.31
13, 19, 20, 23, 25....	5	12.44	11.73	+0.71	+1.11	1.12	+0.30	0.36	+5.84	5.89	+1.58	1.89

TABLE XIII
AVERAGE CENTENNIAL PROPER MOTIONS

Field	Mean Photog. Mag.	No. of Stars	Aver. P.M. in α Alg.	Aver. P.M. in α Arith.	Aver. P.M. in γ Alg.	Aver. P.M. in γ Arith.
S Cassiopeiae.....	8.36	3	-1.43	1.43	-0.07	1.07
V Tauri.....	9.29	4	-1.47	3.40	+1.47	1.89
R Camelopardalis.....	9.77	2	0.00	1.53	+1.53	1.53
R Vulpeculae.....	9.19	3	+3.13	3.13	-0.20	1.50
Weighted mean.....	9.11		-0.06	2.53	+0.68	1.78
S Cassiopeiae.....	10.48	5	-0.57	2.00	-1.29	1.93
U Arietis.....	10.22	6	+ .20	1.87	+0.20	1.53
V Tauri.....	10.46	4	+ .11	1.58	-0.74	1.26
R Camelopardalis.....	11.53	4	+0.23	0.31	-1.92	2.15
R Serpentis.....	10.67	3	+1.01	1.01	+0.48	1.49
R Draconis.....	11.27	3	-1.42	1.72	- .71	2.72
R Vulpeculae.....	10.67	7	+0.72	1.43	-0.20	0.91
R Pegasi.....	10.79	2	+0.05	3.15	-1.00	1.68
Weighted mean.....	10.70		+0.11	1.52	-0.52	1.61
S Cassiopeiae.....	12.32	4	-0.50	1.36	+2.00	2.00
U Arietis.....	12.24	5	+1.33	1.87	-0.67	1.47
U Arietis.....	12.65	3	+1.26	1.26	-1.26	1.26
V Tauri.....	11.81	5	+0.16	2.42	+2.21	2.21
V Tauri.....	12.42	3	-3.68	3.68	+2.52	2.52
R Camelopardalis.....	12.92	5	-0.38	3.68	+0.69	2.38
R Serpentis.....	11.63	5	- .54	0.89	- .48	0.83
R Draconis.....	12.28	2	-0.18	1.06	+ .77	0.77
R Vulpeculae.....	11.97	3	-1.89	1.89	+ .91	1.17
R Pegasi.....	12.51	9	+2.84	3.26	+0.58	1.00
Weighted mean.....	12.28		+0.30	2.29	+0.68	1.54
S Cassiopeiae.....	15.40	5	+0.93	1.14	-1.72	3.22
U Arietis.....	14.82	5	+1.47	1.87	+0.27	1.20
R Camelopardalis.....	14.70	2	-5.82	5.82	+1.92	1.92
R Serpentis.....	14.69	3	-0.48	1.55	-0.77	1.37
R Draconis.....	14.77	5	-1.24	1.30	+2.37	2.37
R Vulpeculae.....	14.97	4	-4.56	4.56	-2.35	2.41
R Pegasi.....	15.03	5	+1.73	3.15	+2.94	2.94
Weighted mean.....	14.95		-0.58	2.54	+0.39	2.28
S Cassiopeiae.....	16.30	5	+0.43	1.72	-5.00	5.00
V Tauri.....	15.99	6	-6.89	6.89	+1.68	1.68
R Camelopardalis.....	16.39	5	-5.06	5.67	+7.97	7.97
R Serpentis.....	15.87	5	-1.96	2.98	-0.65	2.32
R Draconis.....	15.41	5	-0.71	2.31	-0.83	1.36
R Vulpeculae.....	16.01	4	-6.97	6.97	-3.71	3.71
R Pegasi.....	16.19	5	+1.31	3.68	+2.52	3.31
Weighted mean.....	16.38		-2.83	4.31	+0.44	3.56

TABLE XIIIa
(Standard Stars Omitted)

Field	Mean Photog. Mag.	No. of Stars	Aver. P.M. in α Alg.	Aver. P.M. in α Arith.	Aver. P.M. in γ Alg.	Aver. P.M. in γ Arith.	Aver. P.M.
S Cassiopeiae.....	12.08	1	-2"22	2"22	+3"64	3"64	
U Arietis.....	12.45	2	+2.40	2.40	-1.00	2.73	
U Arietis.....	12.65	3	+1.53	1.53	-1.73	1.73	
V Tauri.....	12.36	4	-3.68	3.68	+3.26	3.26	
R Camelopardalis.....	13.08	3	-0.92	5.59	-0.46	2.30	
R Draconis.....	10.36	2	+2.60	2.60	+8.34	8.34	
R Serpentis.....	11.54	3	-0.12	2.08	-0.65	1.55	
R Vulpeculae.....	10.19	2	+3.00	4.11	-0.78	1.69	
R Pegasi.....	12.44	5	+5.84	5.89	+1.58	1.89	
Weighted mean.....	12.06		+1.19	3.68	+1.17	2.74	4.59

TABLE XIV
SUMMARY

Mean Photog. Mag.	No. of Stars	Aver. P.M. in x Alg.	Aver. P.M. in x Arith.	Aver. P.M. in y Alg.	Aver. P.M. in y Arith.	Aver. P.M. 100 Yr.
9.11.....	10	-0".06	2".53	+0".68	1".78	3".09
10.70.....	34	+ .11	1.52	- .52	1.61	2.22
12.28.....	44	+ .30	2.29	+ .68	1.54	2.75
14.95.....	29	-0.58	2.54	+ .39	2.28	3.41
16.38.....	35	-2.83	4.31	+0.44	3.56	5.59
12.06.....	25	+1.19	3.68	+1.17	2.74	4.59
Without standards....	152					

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END OF VOLUME IV

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